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OF
REINFORCED CONCRETE

FUNDAMENTALS
FORMULÆ—TABLES—EXAMPLES

UNIV. OF
CALIFORNIA

BY

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FOREWORD.

This handy up-to-date little book on Reinforced Concrete will be found of great use, and should be welcomed by all those engaged in Modern Building and Engineering Construction. It is written by Messrs. Siacci and Crookes, and is published by Leightons Ltd., of Auckland.

By the index everyone can see that it contains in a comparatively small space the most important sections of Reinforced Concrete occurring in Building Construction. Every section is dealt with briefly, yet in a very simple and practical manner through tables and numerical examples.

Particularly interesting is the last section dealing with the American flat slab, which system is relatively new in Australasia.

The designer will find in this publication the guide he needs to solve *without hesitation* the main problems of Reinforced Concrete. The book is no doubt of practical assistance to Architects as well as to Engineers, where they are required to design their own structures; it should find a place on the desk of every one of them.

DANIEL B. PATTERSON,
Registered Architect
AUCKLAND (N.Z.).

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PREFACE

This small book contains the important essentials of many larger ones. It is intended to guide and assist Structural Engineers and Architects in the design of buildings and other structures in Reinforced Concrete.

It explains in simple and concise language how problems, which would take hours of labour to solve by using a treatise, may be solved in a few minutes with the maximum permissible accuracy. Some of the tables, in fact, will almost supply the designer with the dimensions of each element of a structure, whilst others will provide the means of making easy and rapid calculations for the same.

Useful data and many worked examples are also given throughout the volume. Amongst the subjects which receive special attention may be mentioned the *American Flat Slab Construction*, which for a large class of buildings has almost superseded in America the old beam and girder design.

The book is intended to be of special value to the non-mathematical man. In this respect it should be appreciated by those professional men whose opportunities have been such, that they have not been able to acquire a knowledge of advanced mathematics.

G.S.
S.I.C., Jr.

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CORRIGENDA.

Page 19—Fourth line from foot—Correct steel to concrete.

Page 23—Formula (13)—Change both multiplication signs to plus.

Page 27—Middle of page—Correct $f_c \times kd$ to $f_s \times kd$

Page 30—Near foot of page—Correct 4th, 2nd, 3rd to 2nd, 3rd, 4th

Page 46—First line—Correct (Type F) to (Type D)

Page 50—Seventh line down—Correct $\frac{1}{10} \times 12^2 \times 12$ to $\frac{1}{10} \times 300 \times 12^2 \times 12$

ESTIMATING OF QUANTITIES OF MATERIALS

For estimating quantities and measuring the ingredients the following table is given for three different mixtures of concrete :—

Mixture	Materials	QUANTITIES.		
		For 1 cub. yd. of Concrete	For 1 bag of Cement	For 1 batch or for 1 barrel of Cement
1 : 2 : 4	Cement	1.51 barrels = 568lbs.	1 bag = 124lbs.	1 barrel of 376lbs.
	Sand	0.45 cub. yd.	2.64 cub. ft.	8 cub. ft. (Box 30″ × 30″ × 15⅓″)
	Gravel or Stone	0.89 cub. yd.	5.28 cub. ft.	16 cub. ft. (Box 60″ × 30″ × 15⅓″)
				Above makes 17¾ cub. ft. or .6623 c. yd. of concrete.
1 : 1½ : 3	Cement	1.91 barrels = 718lbs.	1 bag = 124lbs.	1 barrel of 376lbs.
	Sand	0.42 cub. yd.	1.98 cub. ft.	6 cub. ft. (Box 2′ × 3′ × 12″)
	Gravel or Stone	0.85 cub. yd.	3.96 cub. ft.	12 cub. ft. (Box 4′ × 3′ × 12″)
				Above makes 14⅛ cub. ft. or .5255 c. yd. of concrete.
1 : 1 : 2	Cement	2.63 barrels = 989lbs.	1 bag = 124lbs.	1 barrel of 376lbs.
	Sand	0.39 cub. yd.	1.32 cub. ft.	4 cub. ft. (Box 2′ × 2′ × 12″)
	Gravel or Stone	0.78 cub. yd.	2.64 cub. ft.	8 cub. ft. (Box 4′ × 2′ × 12″)
				Above makes 10¼ cub. ft., or .3802 c. yd. of concrete.

NOTE.—There are assumed to be 18 bags of cement to the ton, and the weight of cement is taken as 94lbs. per cubic foot.

The proportions and quantities are based on average conditions of gravel or stone, with 45 per cent. voids. For carefully graded aggregate a smaller proportion of cement is required.

ULTIMATE STRENGTH AND WORKING STRESSES FOR CONCRETE AND STEEL.

In reinforced concrete calculations it is common practice to make use of definite working stresses, based on the ultimate strength of concrete per square inch after 28 days. For an aggregate like gravel, hard limestone or hard sandstone, the following table gives the ultimate strength of concrete.

Ultimate Strength of Concrete in Lbs. Per Square Inch.

Mixture	After 28 days	After 3 months	After 6 months	After 1 year
1 : 2 : 4	2000	2700	3100	3550
1 : 1½ : 3	2500	3375	3875	4435
1 : 1 : 2	3000	4050	4650	5325

In reinforced concrete construction, a concrete mixture 1 : 2 : 4, whose ultimate strength after 28 days is 2000lbs. per sq. inch, is generally used. For columns and watertight structures, like reservoirs, richer mixtures are often preferred.

Working Stresses for a 2000lb. Concrete (1:2:4).

KIND OF STRESS.	Percentage of ultim. strength	Working Stress in lbs. per sq. in.
BEARING (such as bearing on conc. foundns.)	32.5	f_c = 650
AXIAL COMPRESSION (columns)	22.5	f_c = 450
COMPRESSION IN EXTREME FIBRE (beams)	32.5	f_c = 650
COMPRESSION IN EXTREME FIBRE (at the sup't.)	37.5	f_c = 750
SHEAR (punching shear)	6	v = 120
SHEAR (for beams with full web reinf.)	6	v = 120
SHEAR (for beams without full web reinf.)	2	v = 40
BOND (plain bars)	4	u = 80
BOND (deformed bars)	5	u = 100

Strength and Working Stress of Medium Steel.

Ultimate tensile strength = 55,000 to 70,000 lbs. per sq. inch
Yield point = 33,000 lbs. per sq. inch
Working stress = f_s = 16,000 lbs. per sq. inch

Ratios of Moduli of Elasticity. $E_s / E_c = n$.

For 1 : 2 : 4 mixture (2000 lb. concrete) $n = 15$
For 1 : $1\frac{1}{2}$: 3 mixture (2500 lb. concrete) $n = 12$
For 1 : 1 : 2 mixture (3000 lb. concrete) $n = 10$

Bearing Capacity of Soils.

Nature of Ground.	Tons.
Sound limestone or sandstone (compact beds)	20 p. sq. ft.
Sound shale or soft friable rock	8 p. sq. ft.
Gravel and sand compact, or dry and coarse gravel	6 p. sq. ft.
Hard clay, dry and in thick beds	4 p. sq. ft.
Sand and clay mixed, or confined sand	2 p. sq. ft.
Soft clay or wet loose sand	1 p. sq. ft.

Weight of Materials.

Material.	Weight.
Gravel or hard stone concrete	150 lbs. p. cu. ft.
Steel	490 lbs. p. cu. ft.
Cement	94 lbs. p. cu. ft.
Water	62.5 lbs. p. cu. ft.
Dry Sand	100 lbs. p. cu. ft.
Brickwork in cement mortar	115 lbs. p. cu. ft.
Terra-cotta (hollow), for partitions	60 to 70 lbs. p. cu. ft.
Terra-cotta (solid), for partitions	120 lbs. p. cu. ft.

Proportioning of Aggregates.

The strongest concrete is the densest. Its strength varies directly as the amount of cement, and inversely as the percentage of voids in the mixture of aggregates. Thus for a fixed, or given, quantity of cement the strongest concrete is obtained when the fine and the coarse materials are so well proportioned, that the voids in the mixture are reduced to a minimum. An excess of fine material must, however, be avoided.

Including the cement the total amount of fine material must be sufficient to fill the voids.

Voids in coarse aggregate may be determined closely by measuring the amount of water required to fill the voids in a given volume of aggregate. For average conditions and for broken stone with dust screened out, the percentage of voids is about 45.

NOTATION.

(See Figures 1 to 4).

The following notation is in accordance with standard American practice :—

f_s = Unit tensile stress in steel.

f_c = Unit compressive extreme fibre stress in concrete.

n = Ratio E_s / E_c

M_s = Resisting moment as determined by steel.

M_c = Resisting moment as determined by concrete.

A_s = Area of steel in tension (or a_s)

A'_s = Area of steel in compression (or a'_s).

M = Bending moment.

b = Breadth of rectangular beam or breadth of flange of T-beam

b' = Breadth of web of T-beam.

z = Depth from compression surface of beam to resultant of compressive stresses.

d = Depth of beam, or distance from top of beam to axis of steel in tension.

kd = Depth of neutral axis of a section below top of beam.

jd = Arm of resisting couple.

p = Ratio corresponding to percentage of steel in tension $= A_s / bd$.

p' = Ratio corresponding to percentage of steel in compression $= A'_s / bd$.

Tables I. and II., see below, are given to enable the designer to decide on the best arrangement of reinforcement as regards number, area, size, and spacing of bars. They are also useful for estimating the weight of steel since the weight per linear foot for one foot width is given for various bar spacings.

Table I.

FOR ARRANGEMENT OF STEEL IN COLUMNS, BEAMS, SLABS, ETC.

Diam. of Rod.	Circumference. ins.	Weight per ft. lbs.	Sectional Area of Steel Rods in Number:—											
			1	2	3	4	5	6	7	8	9	10	11	12
1/4″	0.785	0.17	.049	.098	.147	.196	.246	.294	.344	.393	.442	.491	.540	.589
5/16″	0.982	0.26	.077	.153	.230	.307	.384	.460	.537	.614	.690	.767	.844	.920
3/8″	1.178	0.38	.110	.221	.331	.442	.552	.662	.773	.883	.994	1.10	1.21	1.32
7/16″	1.374	0.51	.150	.301	.451	.601	.751	.902	1.05	1.20	1.35	1.50	1.65	1.80
1/2″	1.571	0.67	.196	.393	.589	.785	.982	1.18	1.37	1.57	1.77	1.96	2.16	2.36
9/16″	1.767	0.85	.249	.497	.745	.994	1.24	1.49	1.74	1.99	2.24	2.48	2.73	2.98
5/8″	1.964	1.04	.307	.613	.920	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68
11/16″	2.160	1.26	.371	.742	1.11	1.48	1.86	2.23	2.60	2.97	3.34	3.71	4.08	4.45
3/4″	2,356	1.50	.442	.883	1.33	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30
7/8″	2.749	2.04	.601	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22
1″	3.142	2.67	.785	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43
1⅛″	3.534	3.38	.994	1.99	2.98	3.98	4.97	5.96	6.96	7.95	8.95	9.94	10.9	11.9
1¼″	3.927	4.17	1.23	2.45	3.68	4.91	6.14	7.36	8.59	9.82	11.0	12.2	13.5	14.7

Table II.

ARRANGEMENT OF STEEL IN SLABS: AREAS AND WEIGHTS FOR GIVEN SPACINGS.

	Secnl. Area of Rods p. ft. width of slab.							Weight of Rods p. lin. ft. p. ft. width of slab.						
Spacing	1/4″	5/16″	3/8″	7/16″	1/2″	9/16″	5/8	1/4″	5/16″	3/8″	7/16″	1/2″	9/16″	5/8″
3″	.196	.307	.441	.601	.785	.994	1.23	.666	1.04	1.50	2.04	2.67	3.38	4.17
3½″	.168	.263	.378	.515	.673	.852	1.05	.568	.895	1.29	1.76	2.29	2.90	3.58
4″	.147	.230	.331	.451	.589	.745	.920	.500	.783	1.13	1.53	2.00	2.54	3.19
4½″	.131	.204	.294	.401	.523	.663	.818	.444	.696	1.00	1.36	1.78	2.25	2.78
5″	.118	.184	.265	.361	.471	.596	.736	.400	.626	.900	1.23	1.60	2.03	2.50
5½″	.107	.167	.240	.327	.428	.542	.669	.364	.570	.818	1.12	1.46	1.83	2.28
6″	.098	.153	.221	.300	.392	.497	.613	.333	.522	.750	1.02	1.33	1.69	2.09
6½″	.090	.141	.204	.277	.362	.458	.566	.307	.482	.693	.944	1.23	1.56	1.93
7″	.084	.131	.189	.257	.336	.426	.526	.285	.447	.643	.876	1.14	1.45	1.79
7½″	.078	.123	.176	.240	.314	.397	.493	.266	.418	.600	.818	1.07	1.35	1.67
8″	.073	.115	.165	.225	.294	.373	.460	.250	.392	.563	.767	1.00	1.27	1.57

SIMPLE BENDING AND SIMPLE REINFORCEMENT.

Formulæ.

Figure I. represents a rectangular beam and its stress diagram. The portion above the neutral axis is in compression, and has an average working stress of $\frac{1}{2} f_c$ lbs. p. sq. in. Its total stress is $\frac{1}{2} f_c \times kd$, in which k is the ratio of the depth of the neutral axis below the top of the beam to d.

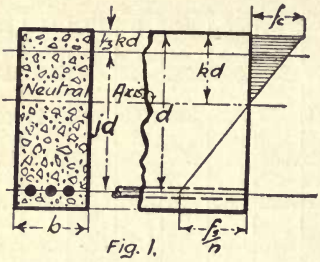


Fig. I.

The portion of the concrete below the neutral axis is stressed in tension. This is neglected in calculations, so that for simple bending the total tension in the steel must equal the total compression,

$$\therefore \quad \tfrac{1}{2} f_c \times kbd \quad = \quad f_s \times A_s$$

By introducing the percentage of steel in tension $p = A_s / bd$ we obtain :—

$$f_s \, p \quad = \quad \tfrac{1}{2} f_c \, k \qquad \text{- - - - - - - - - -} \qquad (1)$$

A second relationship between the working stress in steel and concrete and the depth of the neutral axis is obtained by assuming that the deformation of the fibres vary as their distances from the neutral axis :—

$$\frac{f_s}{n f_c} = \frac{d - kd}{kd} \qquad \text{- - - - - - - - - -} \qquad (2)$$

The arm of the resisting couple, $jd = d - \frac{1}{3} kd$ or

$$j \quad = \quad 1 - \tfrac{1}{3} k \qquad \text{- - - - - - - - - - -} \qquad (3)$$

The safe resisting moment as determined by the steel is :—

$$M_s \quad = \quad f_s \, A_s \, jd \quad = \quad f_s \, p \, j \, bd^2, \text{ or,}$$

$$M_s \quad = \quad K \, bd^2 \qquad \text{- - - - - - - - -} \qquad (4)$$

$$\text{in which } K \quad = \quad p . j . f_s \qquad \text{- - - \quad - - -} \qquad (5)$$

The safe resisting moment as determined by the concrete is :—

$$M_c \quad = \quad \tfrac{1}{2} f_c \, kj \, bd^2, \text{ or,}$$

$$M_c \quad = \quad f_c \, bd^2 \times \frac{1}{C} \qquad \text{- - - - - - -} \qquad (6)$$

$$\text{in which} \quad C \quad = \frac{2}{kj}$$

From the preceding formulæ we derive the following :—

$$p \; = \; \frac{\tfrac{1}{2}}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{n f_c} + 1\right)} \qquad \text{- - - - - - - - -} \qquad (7)$$

$$\text{and } k \; = \; \sqrt{2 \, pn + (pn)^2} - pn \qquad \text{- - - - - - - - -} \qquad (8)$$

These complete the set to be used in the calculation of rectangular beams and slabs with no compressive reinforcement.

Table III., below, gives the values of k, j, p, and K, for $f_s = 16{,}000$ lbs. p. sq. in., and for different values of f_c and n.

Table III.

f_s	f_c	$n = 12$				$n = 15$			
		k	j	p	K	k	j	p	K
16,000	500	0.273	0.909	0.0043	62	0.319	0.894	0.0050	71
16,000	550	0.292	0.903	0.0050	72	0.339	0.887	0.0058	82
16,000	600	0.310	0.897	0.0058	83	0.358	0.881	0.0067	94
16,000	650	0.328	0.891	0.0067	95	0.378	0.874	0.0077	107
16,000	700	0.344	0.885	0.0075	106	0.397	0.868	0.0087	121
16,000	750	0.360	0.880	0.0084	119	0.414	0.862	0.0097	134

Table III., above, has been computed by using formulæ (3), (5), (7), and (8), which apply to rectangular beams with simple reinforcement. They can, however, also be used for beams reinforced in compression, provided that the steel in compression is placed very near to the centroid of the compressive stresses, viz., at a distance from the compressive face of the beam of $z = \frac{1}{3}kd$.

For the design of Rectangular Beams **Table IV.** has been computed.

Table IV.

$\frac{f_s}{f_c}$	k	j	C	$\frac{f_s}{f_c}C$	$\frac{f_s}{f_c}$	k	j	C	$\frac{f_s}{f_c}C$
8	0.652	0.783	3.92	31	28	0.349	0.884	6.49	182
10	0.600	0.800	4.17	42	30	0.333	0.889	6.75	203
12	0.556	0.815	4.42	53	32	0.319	0.894	7.01	224
12.5	0.549	0.817	4.49	56	34	0.306	0.898	7.27	248
13	0.536	0.821	4.54	59	36	0.294	0.902	7.55	271
13.5	0.526	0.825	4.61	62	38	0.283	0.906	7.81	296
14	0.517	0.828	4.67	65	40	0.273	0.909	8.07	323
14.5	0.509	0.830	4.74	68	42	0.263	0.912	8.33	350
15	0.500	0.833	4.80	72	44	0.254	0.915	8.61	378
15.5	0.491	0.836	4.87	75	46	0.246	0.918	8.85	408
16	0.484	0.839	4.93	79	48	0.238	0.921	9.13	438
17	0.469	0.844	5.05	86	50	0.231	0.923	9.39	470
18	0.455	0.848	5.19	93	52	0.224	0.925	9.66	502
19	0.441	0.853	5.32	101	54	0.217	0.928	9.92	536
20	0.429	0.857	5.44	109	56	0.211	0.930	10.18	570
21	0.417	0.861	5.57	117	58	0.205	0.932	10.46	606
22	0.405	0.865	5.68	125	60	0.200	0.933	10.71	643
23	0.395	0.868	5.83	134	62	0.195	0.935	10.99	681
24	0.385	0.872	5.95	143	64	0.190	0.937	11.25	720
25	0.375	0.875	6.10	152	66	0.185	0.938	11.51	760
26	0.366	0.878	6.23	162	68	0.181	0.940	11.78	801
27	0.357	0.881	6.35	172	70	0.177	0.941	12.01	841

Formulæ (2), (3), and (6), have been used in the calculation of **Table IV.** Formulæ (6) has been put in the form :—

$$\frac{bd^2 f_s}{M} = \frac{f_s}{f_c} \times C. \quad \text{- - - - - - - - - -} \quad (6a)$$

by which we can easily find out if the working stress in the concrete in a beam of given dimensions is beyond the limit allowed, and consequently the area (if any) of steel to be added in compression to reduce such working stress.

Example :—ILLUSTRATING THE APPLICATION OF **Table IV.**

A concrete beam is subject to a bending moment,

$$M = 358{,}000 \text{ lb. ins.}$$

The allowable stresses are :—

$$f_s = 16{,}000; \quad f_c = 600 \text{ and } n = 15.$$

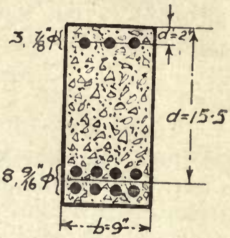


Fig. 2.

Assume that the breadth of the beam $b = 9''$ and that the depth, $d = 15\frac{1}{2}''$.

$$\frac{bd^2 f_s}{M} = \frac{9 \times 15.5 \times 15.5 \times 16000}{358{,}000}$$

$$= 98 = \frac{f_s}{f_c} \times C.$$

For the nearest value to this Table IV. gives :—

$k = 0.441$ (whence $kd = 0.441 \times 15.5 = 6.85$)
$j = 0.853$ (whence $jd = 0.853 \times 15.5 = 13.22$)
$\frac{f_s}{f_c} = 19$

Hence, Steel area in tension, $a_s = \frac{358{,}000}{13.22 \times 16{,}000} = 1.7$ sq. ins.

Say 8 rods 9/16″ diam., whose area is 1.99 sq. ins. (which is greater than 1.7 sq. ins).

Working stress of concrete $f_c = f_s/19 = 16000/19$
$= 840$ lbs. p. sq. in.

This is greater than the limit which is 600 lbs. p. sq. in.

The high stress in the concrete can be reduced in one of the following ways :—

(a) By increasing the breadth of the beam, *b*.
(b) By increasing the depth of the beam, *d*.
(c) By increasing both dimensions, *b* and *d*.
(d) By adding reinforcement in compression.

Supposing that we decide in favour of the additional reinforcement in compression, we proceed as follows:—

The external compression $= M/jd = 358{,}000 \div 13.22 =$ 27,080 lbs.
The available compression $= b \times kd \times \frac{1}{2}f_c = 9 \times 6.85 \times \frac{1}{2}600 =$ 18,548 lbs.
The excess which has to be resisted by steel in compression = 8,532 lbs.

The allowable unit compressive stress on steel placed at a distance $d' = 2''$ from the top of the beam (i.e. very near to the centroid of the compression area, $\frac{1}{3}kd$ below top of beam) is

$$nf_c \times (kd-d')/kd = 15 \times 600 \times (6.85-2) \div 6.85$$
$$= 6300 \text{ lbs. p. sq. in.}$$

Hence the area of steel required in compression

$$= 8532 \div 6300 = 1.35 \text{ sq. ins.}$$

Say 3 rods $\frac{7}{8}''$ diam. whose area is 1.80 sq. ins. (which is greater than 1.35 sq. in.).

II. RECTANGULAR BEAMS.

Simple Bending and Double Reinforcement.

Formulæ:—

With the object of simplifying the general formulæ for rectangular beams with double reinforcement, which are too cumbersome for practical use, we shall assume that the steel in compression is placed at the centroid of the compression area of the concrete, so that,

$$z = \tfrac{1}{3}kd \text{ and } j = 1 - \tfrac{1}{3}k \text{ - - - - - - - - - } (8)$$

With this assumption the formulæ for equilibrium are:—

$$6\,np\,(1-k) = 3k^2 + 4\,np'k \text{ - - - - - - - - } (9)$$
$$\text{and } M_c = (\tfrac{1}{2}k + \tfrac{2}{3}np')\,jf_c \times bd^2$$

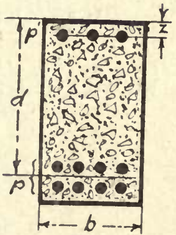


Fig. 3.

Introducing the abbreviation,

$$L = (\tfrac{1}{2}k + \tfrac{2}{3}\,np')\,j \text{ - - - - } (10)$$

$$\text{we obtain } L = \frac{M}{bd^2 \times f_c} \text{ - - - - - - - - - } (11)$$

The value of k to be computed in formula (10) is given by the formulæ:—

$$k = -(np + \tfrac{2}{3}np') + \sqrt{(np + \tfrac{2}{3}np')^2 + 2np} \text{ - - - - - - } (12)$$

which is obtained by solving formula (9) for k.

The set of formulæ (8), (9), (10), (11), (12) can be conveniently used in the calculation of rectangular beams with compressive reinforcement. By them we have computed **Table V.**, which supplies immediately the values of k, j, L, and $p' = A_s'/bd$, for different values of $p = A_s/bd$. Hence by using **Table V.** the designer will have the choice of taking from amongst the various solutions, the one which appears to be best suited to any particular case. This table is also useful for reviewing a given rectangular beam.

After having computed the value of

$$L = \frac{M}{bd^2 f_c}$$

for the assumed dimensions b and d, and for a working stress of, say, 600 lbs. p. sq. in. for the concrete, we find in **Table V.** the various possible solutions for p and p', including $p = o$ (viz., with no compressive reinforcement).

The steel area in tension will then be $A_s = pbd$.
The steel area in compression ,, ,, ,, $A'_s = p'bd$.
The working stress of the steel ,, ,, ,, $f_s = M/A_s jd$.

Table V.

DESIGN AND REVIEW OF RECTANGULAR BEAMS WITH DOUBLE REINFORCEMENT.

p	$p' = 0$			$p' = 0.20p$			$p' = 0.40p$		
	k	j	L	k	j	L	k	j	L
0.005	0.319	0.894	0.142	0.312	0.896	0.148	0.304	0.899	0.153
0.006	0.344	0.886	0.152	0.335	0.888	0.160	0.326	0.891	0.166
0.007	0.366	0.878	0.161	0.354	0.882	0.169	0.344	0.886	0.177
0.008	0.384	0.872	0.168	0.372	0.876	0.176	0.361	0.880	0.187
0.009	0.402	0.866	0.174	0.389	0.870	0.184	0.376	0.875	0.196
0.010	0.418	0.861	0.180	0.403	0.866	0.192	0.390	0.870	0.204
0.011	0.433	0.856	0.185	0.417	0.861	0.199	0.402	0.866	0.212
0.012	0.446	0.851	0.190	0.430	0.857	0.205	0.414	0.862	0.220
0.013	0.459	0.847	0.194	0.443	0.852	0.211	0.427	0.858	0.228
0.014	0.471	0.843	0.199	0.452	0.849	0.216	0.435	0.855	0.234
0.015	0.482	0.839	0.202	0.462	0.846	0.220	0.443	0.852	0.239
0.020	0.530	0.823	0.218	0.506	0.831	0.243	0.483	0.839	0.269

p	$p = 0.60p$			$p' = 0.80p$			$p' = p$		
	k	j	L	k	j	L	k	j	L
0.005	0.296	0.901	0.159	0.289	0.904	0.166	0.282	0.906	0.172
0.006	0.317	0.894	0.175	0.308	0.897	0.181	0.300	0.900	0.189
0.007	0.334	0.889	0.186	0.325	0.892	0.195	0.316	0.895	0.204
0.008	0.350	0.883	0.197	0.339	0.887	0.208	0.329	0.890	0.217
0.009	0.364	0.879	0.207	0.352	0.883	0.220	0.341	0.886	0.230
0.010	0.377	0.874	0.217	0.364	0.879	0.231	0.352	0.883	0.244
0.011	0.388	0.871	0.227	0.375	0.875	0.242	0.362	0.879	0.256
0.012	0.399	0.867	0.236	0.385	0.872	0.252	0.371	0.876	0.268
0.013	0.411	0.863	0.245	0.395	0.869	0.262	0.379	0.874	0.279
0.014	0.418	0.861	0.253	0.402	0.866	0.271	0.387	0.871	0.290
0.015	0.425	0.858	0.259	0.410	0.863	0.280	0.394	0.869	0.301
0.020	0.461	0.846	0.296	0.441	0.853	0.324	0.422	0.859	0.353

Examples:—

(1). Find the dimensions of a rectangular beam given that—

f_s = 16000 lbs. p. sq. in.
f_c = 600 lbs. p. sq. in.
n = 15.

Bending Moment = 358,000 lb. ins.

Assuming that the breadth of the beam will be 9″ and that its depth will be 15½″ we find that—

$$L = \frac{358,000}{9 \times 15.5 \times 15.5 \times 600} = 0.275.$$

From among the solutions offered by **Table V.** we choose the symetrical reinforcement ($p = p'$), which is the most economical. The nearest value of L in the table is, L = 0.279 for which

$$p = p' = 0.013.$$

Hence $a_s = a'_s = 0.013 \times 9 \times 15.5.$
$= 1.8$ sq. ins.

The reinforcement used will be—

In tension 8 rods 9/16″ diam. (area 1.99 sq. ins., which is greater than 1.8 sq. ins.)

In compression 3 rods 7/8″ diam. (area 1.8 sq. ins. as required).

---

(2) Given the same beam and the same bending moment as in Example 1, find the working stresses in the steel and the concrete.

$b = 9''$; $d = 15.5''$; $p = p' = 1.80/9 \times 15.5 = 0.013$ (approx.)

Hence, from **Table V.** :—

$j = 0.874$ (therefore $jd = 0.874 \times 15.5 = 13.54$ ins.)
and $L = 0.279$

Therefore, Working stress of steel $f_s = \dfrac{M}{a_s \times jd} = \dfrac{358,000}{1.80 \times 13.54}$
$= 14,700$ (less than 16,000 limit)

Working stress of steel $f_c = \dfrac{M}{L \times bd^2} = \dfrac{358,000}{0.279 \times 9 \times 15.5 \times 15.5}$
$= 597$ (less than 600 limit)

When a rectangular beam is not limited as to size it is more economical to let compressive stresses be carried by concrete only.

Double Reinforcement is often needed to increase the compression area on the compression side of a beam, when a rectangular beam is limited as to size. Such reinforcement is nearly always required at the support of continuous or restrained beams, where, the bending moment being negative, the area of conrete in compression is merely that provided by the web of the beam below the neutral axis. This area is usually small and insufficient in spite of the fact that a higher working stress is allowable for the concrete at the support. For a 1 : 2 : 4 : mixture this stress is commonly taken at 750lbs. per sq. in. instead of 600 to 650lbs. per. sq. in. as at the middle of the span.

The Width of a rectangular beam should not be less than 1/25th of its length to prevent side bending, unless it is supported laterally by cross-beams or by a slab.

The Depth of a rectangular beam generally requires two or three trials for its determination. For uniformly distributed loads 1/12th of its length is often a convenient depth, whilst for a concentrated load at mid-span a depth equal to 1/6th of the length should not be exceeded.

III. SLABS.

For slabs the formulæ to be used are those for rectangular beams simply reinforced, (see Section I.),in which b is taken as 12″ and K is given by **Table III.** For a 1 : 2 : 4 mix of concrete the allowable stresses in slabs are :—

$$f_s = 16{,}000;\ f_c = 600;\ \text{and}\ n = 12.$$

For these values **Table III.** gives :—

$$K = 83 \text{ and } p = 0.0058.$$

Inserting these in formulæ (4) we obtain :—

$$\text{Depth of slab, } d = \sqrt{\frac{M}{83 \times 12}} = 0.0316\ \sqrt{M}$$

$$\text{The steel area, } a_s = p\ b\ d = 0.0058 \times 12 \times d = 0.0696d.$$

Table VI. has been computed by using these two simple formulæ. This table gives at once the size of slab, the steel area required, and alternative reinforcement for bending moments from 6,000 up to 35, 000 lb. ins.

Table VI.

FOR DESIGN OF SLABS.

$f_c = 600$; $f_s = 16{,}000$; $n = 12$.

M = Resisting Moment of slab one foot wide.

M lb. ins	d ins.	t ins.	a_s sq. ins.	Alternative Reinforcement.
6,000	2.5	3.5	0.176	1/4″ spacing 3″ or
7,000	2.7	3.5	0.188	5/16″ ,, 4½″
8,000	2.8	4.0	0.195	5/16″ ,, 4″ or
9,000	3.0	4.0	0.210	3/8″ ,, 6″
10,000	3.2	4.0	0.223	5/16″ ,, 3½″ or
11,000	3.4	4.5	0.236	3/8″ ,, 5½″
12,000	3.5	4.5	0.244	5/16″ ,, 3½″ or
13,000	3.7	4.5	0.257	3/8″ ,, 5″
14,000	3.8	5.0	0.265	5/16″ ,, 3″ or
15,000	3.9	5.0	0.271	3/8″ ,, 4½″
16,000	4.0	5.0	0.278	5/16″ ,, 3″ or
17,000	4.2	5.0	0.292	3/8″ ,, 4½″
18,000	4.3	5.5	0.299	3/8″ ,, 4″ or
19,000	4.4	5.5	0.306	7/16″ ,, 5½″
20,000	4.5	5.5	0.314	3/8″ ,, 4″ or
21,000	4.6	5.5	0.320	7/16″ ,, 5½″
22,000	4.7	5.5	0.327	3/8″ ,, 3½″ or
23,000	4.8	6.0	0.334	7/16″ ,, 5″
24,000	4.9	6.0	0.341	3/8″ ,, 3½″ or
25,000	5.0	6.0	0.348	7/16″ ,, 5″
26,000	5.1	6.0	0.354	3/8″ ,, 3½″ or
27,000	5.2	6.0	0.362	7/16″ ,, 5″
28,000	5.3	6.5	0.369	3/8″ ,, 3½″ or
29,000	5.4	6.5	0.375	7/16″ ,, 4½″
30,000	5.5	6.5	0.383	3/8″ ,, 3″ or
31,000	5.6	6.5	0.390	7/16″ ,, 4½″
32,000	5.7	6.5	0.396	3/8″ ,, 3″ or
33,000	5.8	7.0	0.404	7/16″ ,, 4″
34,000	5.9	7.0	0.410	7/16″ ,, 4″ or
35,000	6.0	7.0	0.416	1/2″ ,, 5½″

Numerical Example:

Determine the size and steel area for a slab which is subjected to a bending moment of 10,000 lb. ins.

M = 10,000 lb. ins. For this value of M, **Table VI.** gives:—

(1) Thickness of slab, $t = 4''$, and depth, $d = 3.2''$.
(2) Steel area, $a_s = 0.223$ sq. ins.
(3) Reinforcement required = 5/16″ diam. rods at 3½″ spacing, or 3/8″ diam. rods at 5½″ spacing.

Supposing we want to increase the thickness of the slab by, say, ½″. The effective depth d, will then be 3.7″.

The steel area required will therefore be—

$$a_s = \frac{3.2}{3.7} \times 0.223$$
$$= 0.19 \text{ sq. in.}$$

The working stress of the concrete would be—

$$f_c = \frac{3.2}{3.7} \times 600$$
$$= 520 \text{ lbs. per sq. in.}$$

VALUE OF n USED IN SLAB CALCULATIONS.

It is clear that if we had assumed a ratio of moduli $n = 15$, instead of $n = 12$, the percentage of steel required would have been $p = 0.0067$, which is a little more than is required for $n = 12$, but the thickness of the slab would have been a little less. Consequently, by using $n = 12$ we have a greater safe resisting moment for the concrete, which is very desirable for slabs.

IV.—T-BEAMS.

Formulæ:

Figure 4 represents a T-beam and its stress diagram. Unless the flange is very thick the neutral axis lies in the web. Neglecting the compression between the bottom of the slab and the neutral axis, which is very small compared with the area of the flange, the area in compression is bt, and

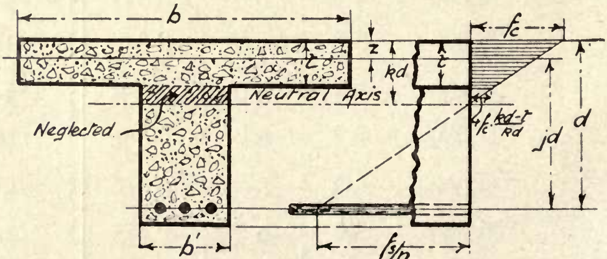


Fig. 4.

$$\text{The total compression} = f_c \frac{2kd - t}{2kd} \times bt.$$

The total tension, which has to be resisted only by the steel $= A_s \times f_s$.

Total tension being equal to total compression—

$$A_s \times f_s = f_c \frac{2kd - t}{2kd} \times bt, \text{ in which}$$

$$k = \frac{n.f_c}{f_s + nf_c}, \text{ as for rectangular beams.}$$

Solving these two equations for *kd*, we obtain the position of the neutral axis:—

$$kd = \frac{2ndA_s \times bt^2}{2n\ A_s \times 2bt} \qquad (13)$$

The arm of the resisting moment is,

$$jd = d - z \qquad (14)$$

$$\text{in which, } z = \frac{3kd - 2t}{2kd - t} \times \frac{t}{3} \qquad (15)$$

For approximate design we assume that the resultant of compression lies in the centroid as for rectangular beams, consequently,

$z = \frac{1}{3}kd$ and $j = 1 - \frac{1}{3}k$.

The safe resisting moment as determined by the steel is,

$$M_s = A_s \times f_s \times jd. \qquad (16)$$

The safe resisting moment as determined by the concrete is,

$$M_c = \frac{2kd - t}{2kd} . bt . jd . f_c \qquad (17)$$

When the neutral axis of a T-beam lies in the web formulæ (13), (14), (15), (16) and (17) should be used in design, but if the neutral axis lies in the flange it should be designed as a rectangular beam. By increasing the depth of a T-beam we can bring down the neutral axis to lie, as is desired, in the web. Table VII. is for use in designing T-beams, and offers the designer more than one solution, according as opportunity offers for the adoption of a certain depth or a certain size with a certain number of reinforcing bars, say 2, 4, or 6 round bars. For the calculation of this table formula (16) has been used, namely,

$$M = a_s \times f_s \times jd.$$

In this it is assumed that,

f_s = 16,000 lbs. per sq. in. and $jd = 8/9d. = 0.889d$.

By assuming $jd = 0.889d$, the resulting values of the resisting moments are sufficiently approximate to the true ones. In fact, although the area in compression has been neglected, the errors involved are small, and are on the side of safety. The error is a maximum when the breadth of the flange is very large. Assuming that it is so large as to reduce the stress in the concrete to, say, 170 lbs. per sq. in., then for $f_s = 16{,}000$ and $n = 12$

$$k = \frac{nf_c}{f_s + nf_c} = 0.113,$$

and $j = 1 - \frac{1}{3}k = 1 - \frac{1}{3}0.113 = 0.962$ (greater than 0.889). Hence the ratio of the two areas of steel is

$$\frac{a_s \text{ from Table VII.}}{a_s \text{ exact}} = \frac{0.962}{0.889}$$

$= 1.08$, say 8% which is of no practical importance, and is on the safe side.

Table VII. — Resistin

f_s = 16,000 lbs. p. sq. in.

(Resisting moments are given in thousands of lb. ins.)

Height of Web. H ins.	Slab Thickness. t ins.	Depth d ins.	Rods $\frac{1}{2}''$ diam : 2			Rods 9/16″ diam : 2			Rods $\frac{5}{8}''$ diam : 2		
			2	4	6	2	4	6	2	4	6
			$a_s =$ .393	$a_s =$ .786	$a_s =$ 1.18	$a_s =$ .497	$a_s =$ .994	$a_s =$ 1.49	$a_s =$ .614	$a_s =$ 1.23	$a_s =$ 1.84
8	4	10	56	112	168	71	141	212	87	175	262
	5	11	61	123	184	78	155	233	96	192	288
	6	12	67	134	202	85	169	254	105	210	314
10	4	12	67	134	202	85	169	254	105	210	314
	5	13	73	146	219	92	184	276	113	227	340
	6	14	78	157	236	99	198	297	122	244	367
12	4	14	78	157	236	99	198	297	122	244	367
	5	15	84	168	253	106	212	318	131	262	393
	6	16	89	178	268	113	226	339	140	279	419
14	4	16	89	178	268	113	226	339	140	279	419
	5	17	95	190	285	120	240	360	148	297	445
	6	18	101	201	302	127	254	382	157	314	471
16	4	18	101	201	302	127	254	382	157	314	471
	5	19	106	213	319	134	268	403	166	332	498
	6	20	112	224	336	141	282	424	174	349	524
18	4	20	112	224	336	141	282	424	174	349	524
	5	21	118	235	353	148	297	445	183	367	550
	6	22	123	246	369	155	311	466	192	384	576
20	4	22	123	246	369	155	311	466	192	384	576
	5	23	129	258	387	162	325	488	201	402	602
	6	24	135	269	404	169	339	509	210	419	629
22	4	24	135	269	404	169	339	509	210	419	629
	5	25	140	280	420	175	353	530	218	437	655
	6	26	146	292	438	184	368	551	227	454	681
24	4	26	146	292	438	184	368	551	227	454	681
	5	27	151	302	454	191	382	573	236	472	707
	6	28	157	314	471	198	396	594	244	489	733
26	4	28	157	314	471	198	396	594	244	489	733
	5	29	162	325	487	205	410	615	253	506	760
	6	30	168	337	504	212	424	636	262	524	786
28	4	30	168	337	504	212	424	636	262	524	786
	5	31	174	348	521	219	438	657	271	542	812
	6	32	179	358	537	226	452	678	279	559	838
30	4	32	179	358	537	226	452	678	279	559	838
	5	33	185	370	555	233	466	700	288	576	865
	6	34	190	380	571	240	480	721	297	594	891

oment of T-Beams.

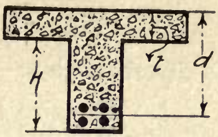


Fig. 5

f_s = 16,000 lbs. p. sq. in.

Resisting moments are given in thousands of lb. ins.)

eight of eb. h ins.	Slab Thickness. t ins.	Depth d ins.	Rods $\frac{3}{4}''$ diam.			Rods $\frac{7}{8}''$ diam.			Rods $1''$ diam.		
			2	4	6	2	4	6	2	4	6
			a_s = .884	a_s = 1.78	a_s = 2.65	a_s = 1.20	a_s = 2.41	a_s = 3.61	a_s = 1.57	a_s = 3.14	a_s = 4.71
8	4	10	126	251	377	171	342	513	224	447	670
	5	11	138	276	415	188	376	565	246	492	737
	6	12	151	302	453	205	411	616	268	536	804
10	4	12	151	302	453	205	411	616	268	536	804
	5	13	164	327	490	222	445	667	290	581	871
	6	14	176	352	528	239	479	719	313	626	938
12	4	14	176	352	528	239	479	719	313	626	938
	5	15	188	377	566	257	513	770	335	670	1005
	6	16	201	402	603	274	548	821	357	715	1073
14	4	16	201	402	603	274	548	821	357	715	1073
	5	17	214	428	641	291	582	873	380	760	1140
	6	18	226	453	679	308	616	924	402	804	1207
16	4	18	226	453	679	308	616	924	402	804	1207
	5	19	239	478	717	325	650	975	425	849	1274
	6	20	251	503	754	342	684	1027	447	894	1340
18	4	20	251	503	754	342	684	1027	447	894	1340
	5	21	264	528	792	359	719	1078	469	938	1408
	6	22	276	553	830	376	753	1129	492	983	1474
20	4	22	276	553	830	376	753	1129	492	983	1474
	5	23	289	578	868	394	787	1181	514	1028	1542
	6	24	302	603	905	411	821	1232	536	1073	1609
22	4	24	302	603	905	411	821	1232	536	1073	1609
	5	25	314	629	943	428	855	1283	559	1117	1676
	6	26	327	654	981	445	890	1334	581	1162	1743
24	4	26	327	654	981	445	890	1334	581	1162	1743
	5	27	339	679	1019	462	924	1386	603	1207	1810
	6	28	352	704	1056	479	958	1437	626	1251	1877
26	4	28	352	704	1056	479	958	1437	626	1251	1877
	5	29	365	729	1094	496	992	1489	648	1296	1944
	6	30	377	754	1132	513	1027	1540	670	1340	2011
28	4	30	377	754	1132	513	1027	1540	670	1340	2011
	5	31	390	780	1169	530	1061	1591	692	1385	2078
	6	32	402	805	1207	548	1095	1642	715	1430	2145
30	4	32	402	805	1207	548	1095	1642	715	1430	2145
	5	33	415	830	1245	565	1129	1694	737	1474	2212
	6	34	428	855	1283	582	1164	1745	760	1519	2279

Determination of the Width of Web of a T-Beam.

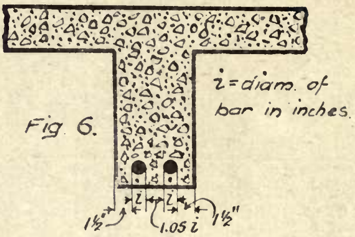


Fig 6.

In a T-beam the width of the web is governed by:—

1. The size and number of rods in tension.
2. The shear stresses.

It is common practice to keep the rods $1\frac{1}{2}''$ clear from the sides of the beam. Between the bars the spacing should be 1.05 x diameter of bar in inches. This minimum spacing has been suggested by Prof. McKibben (U.S.A.) for a working bond stress of 80 lbs. p. sq. in. and a direct working shear stress of 120 lbs. p. sq. in., the concrete mixture being 1 : 2 : 4. The control of the width of the web by the second governing factor, namely, the shear stresses, is indicated by the formula—

$$b' = \frac{V}{v \times jd} \quad \text{- - - - - - - - - - - - - - -} \quad (18)$$

In this formula

V = Total vertical shear.

v = Unit allowable shear stress, which for a 1 : 2 : 4 concrete is 120 lbs. p. sq. in.

Example.—Find the depth of a T-beam to resist a bending moment of 720,000 lb. ins. Determine also the width of the web assuming the total shear is 19,000 lbs.

Table VII. suggests a T-beam whose depth $d = 21''$, with a slab thickness of $5''$ and reinforced with 4 rods $\frac{7}{8}''$ diameter.

The total shear $V = 19{,}000$ lbs.

$$\therefore \text{Width of web, } b' = \frac{V}{v \times jd} = \frac{19{,}000}{120 \times 0.889 \times 21}$$

$$= 9'' \text{ approx. (say } 10''.)$$

With such width the spacing between the rods will be ample.

Supposing the thickness of the slab had to be taken as $4''$ instead of $5''$ then the height of the web would have had to be increased by $1''$ to give the same depth, d, of $21''$.

Determination of Steel Area in Compression, Where Necessary.

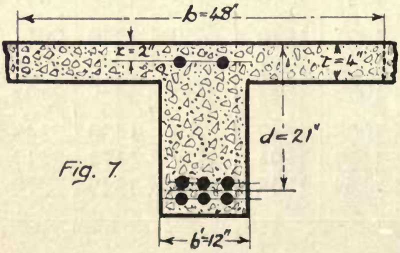


Fig. 7

The procedure is clearly illustrated by the following numerical example.

Let a T-beam be subject to a bending moment = 1,300,000 lb ins.

And let the shear = 25,000 lbs.

The thickness of flange = 4"

The rules regarding the effective width of flange follow this example. Let the span be 16ft. and assume that Rule (1) limits the value of the effective flange width b in this case.

Then $b = \frac{1}{4}$ span $= 48''$

The width of web as determined by shear is, assuming a depth of 21″,

$$b' = \frac{V}{v \times jd} = \frac{25{,}000}{120 \times 0.889 \times 21}$$

$$= 11.1, \text{ say } 12 \text{ inches.}$$

By using **Table VII.**, for the nearest bending moment, $M = 1{,}410{,}000$ lb. ins., we find that the depth of 21″ is suitable with a tensile reinforcement of 6 rods 1″ diameter. It is now necessary to find out whether the working stress of the concrete is within the limit allowed, which, for special reasons, we will assume to be 500 lbs. per sq. in. Neglecting the compression area in the web, the distance of the neutral axis from the top of the beam is given by formula (13),

$$kd = \frac{2nd\,A_s + bt^2}{2n\,A_s + 2bt}$$

$$= \frac{2 \times 15 \times 21 \times 4.71 + 48 \times 4 \times 4}{2 \times 15 \times 4.71 + 2 \times 48 \times 4}$$

$= 7.1''$. This being greater than the slab thickness, $t = 4''$, the neutral axis lies well in the web.

The working stress of the concrete is,

$$f_c = \frac{f_c \times kd}{15\,(d - kd)}$$

$$= \frac{16{,}000 \times 7.1}{15\,(21 - 7.1)}$$

$=$ 544 lbs. p. sq. in. This is greater than our limit of 500 lbs. p. sq. in. consequently compression steel will be necessary.

The total compression $= M/jd = 1{,}300{,}000 \div (0.889 \times 21) = 70{,}000$ lbs.

The total resistance of the concrete $= \frac{1}{2}f_c \,.\, bt \times (2kd - t)/kd.$

$$= \tfrac{1}{2} \times 500 \times 48 \times 4 \times 1.43$$

$$= 68{,}640 \text{ lbs.}$$

The difference of compression which has to be provided for by the steel is evidently $= 70{,}000 - 68{,}640 = 1{,}360$ lbs.

The unit allowable stress on steel placed 2″ below the top of the beam is,

$$f_s' = nf_c \times (kd - z)/kd$$

$$= 15 \times 500 \times (7.1 - 2'') \div 7.1$$

$$= 5400 \text{ lbs. per. sq. in.}$$

Hence the steel area required in compression is,

$$a_s' = 1{,}360 \div 5{,}400.$$

$= 0.252$ sq. ins.; say 2 rods 7/16″ diam. (area 0.301 sq. ins.)

Width of Flange of a T-beam.

The width of the slab which may be considered effective as compression area of a T-beam should be selected in accordance with the following rules. The effective flange width shall in no case exceed :—

(1) One fourth of the span length of the beam.
(2) The clear distance between the beams; and
(3) Its overhanging width on either side of the web shall not be greater than eight times the thickness of the slab.

The above rules conform to the 1921 recommendations of the American Joint Committee.

Provision for Negative Moment at Supports of T-beams.

At supports, where beams are generally to be considered as fixed or partially fixed, the bending moment being negative, the flange is in tension and the lower part of the web in compression. The T-beam must then be designed at supports as a rectangular beam.

The following numerical example will illustrate the method of reviewing a T-beam for negative moment at the support.

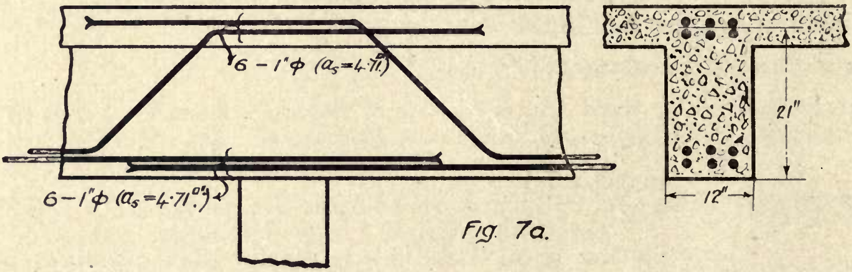


Fig. 7a.

Assume the same beam as in the preceding example and let the bending moment at the support, $M = 1{,}300{,}000$ lb. ins. as in the middle of the span.

Taking into account that one half of the steel in tension, viz., 3 rods 1″ diam. are bent up and carried horizontally over the top of the support, and that from the adjoining span there are 3 more 1″ rods bent up which overlap with the first 3, the reinforcement at the top of the beam over the support will be equal to that in the middle of the span. Sufficient provision is thus made to resist the negative bending moment.

In fact, using **Table V.**,

for $p = p' = 4.71 \div (21 \times 12) = 0.0187$,

we obtain by interpolation,
$j = 0.862$ and $L = 0.336$.

Hence, $f_s = M \div jdA_s$

$$= \frac{1{,}300{,}000}{0.862 \times 21 \times 4.71}$$

$= 15{,}200$ lbs. p. sq. in. (less than 16,000 limit).

$$\begin{aligned} \text{and } f_c &= M \div Lbd^2 \\ &= \frac{1{,}300{,}000}{12 \times 21 \times 21 \times 0.336} \\ &= 730 \text{ lbs. p. sq. in. (less than 750 limit).} \end{aligned}$$

Should the compressive stress in the concrete have been greater than the limit, 750 lbs. p. sq. in., an additional area of steel in compression (placed at the bottom of the web), would have been required. Another method of reducing the stress in the concrete at the support would be to increase the depth of the beam by a flat haunch.

If a haunch is used it must be of such dimensions as to keep the compressive stresses in the web within allowable limits. If no haunch is provided, nor compression steel added, such stresses are liable to be excessive, not only at the face of the support but also for some distance out from it.

It is consequently necessary to determine, as well as the depth, the requisite length of the haunch.

The depth can be best obtained by assuming an increased depth of beam at the support, and then calculating the magnitude of the stresses which will occur if such increased depth is adopted. The depth may be considered as satisfactory if it causes the compressive stress in the concrete to become just under 750 lbs. per sq. in.

The length of the haunch can be determined approximately by using the following formula :—

$$\text{Length of Haunch} = \frac{l}{5} \times \frac{M - M_c}{M}$$

in which,

l = Span of beam

M_c = Resisting moment of the inverted **T**-beam (without the haunch) as determined by the concrete

and M = Negative bending moment at support.

It is assumed in the above formula that the point of inflection occurs at a distance from the support of about one-fifth of the span. This is generally the case. Should the loading be such as to cause the point of inflection to occur at some other point in the span the fraction in the above formula should be altered to correspond.

The above method of designing the haunch applies to exterior as well as to interior spans, when the beam is designed as fully continuous with the exterior column.

At the end of a continuous beam, the provision for negative moment is usually made by the addition of short rods placed at the top over the support. These short rods are bent down at their ends to assist in shear reinforcement.

V.—STIRRUPS IN RECTANGULAR BEAMS AND IN T-BEAMS.

Additional Notation.

V = Total shear.
v = Unit allowable shear stress.
u = Unit allowable bond stress.
i = Diameter of stirrup bars.
s = Spacing of stirrups.

Although the value of stirrups as diagonal tension reinforcement is questionable, their use in beams is made necessary in case of overstressing. They are essential where beams are subjected to dynamic action caused by moving loads. For a web reinforced with stirrups and a 1 : 2 : 4 concrete mixture, v = 120 lbs. per sq. in.

Formulæ.

Total area of prongs of one stirrup,

$$A_s = \frac{\frac{2}{3} V . s}{f_s . jd} \quad \text{- - - - - - - - - - - - - -} \quad (19)$$

1st spacing of stirrups,

$$s = \frac{f_s . A_s . jd}{\frac{2}{3} V} \quad \text{- - - - - - - - - - - - - -} \quad (20)$$

2nd spacing of stirrups = twice the first spacing.
3rd spacing of stirrups = three times the first spacing.

4th spacing of stirrups = four times the first spacing, and so on until such spacing reaches three-quarters of the effective depth of the beam. The latter spacing is the maximum allowed. The above rule for spacing is not based on theoretical formulæ. It is, however, practical, and corresponds generally with calculations. As a rule stirrups should be made with hooked ends to enable them to develop their stress in bond.

Numerical Example:

The maximum vertical shear at the support of a T-beam is 25,000 lbs. The effective depth is 21″, and the breadth of the web is 12″. Find the area and spacing of the stirrups.

$V = 25{,}000$ lbs.; $b' = 12''$; $d = 21''$.

Hence the intensity of shear in the web,

$$\begin{aligned} v &= V / b'jd \\ &= 25000 \div (12 \times 0.889 \times 21) \\ &= 111 \text{ lbs. p. sq. in. (less than 120 limit)} \end{aligned}$$

Using $\frac{3}{8}''$ round rods to form two prong stirrups the area, A_s = 0.22,

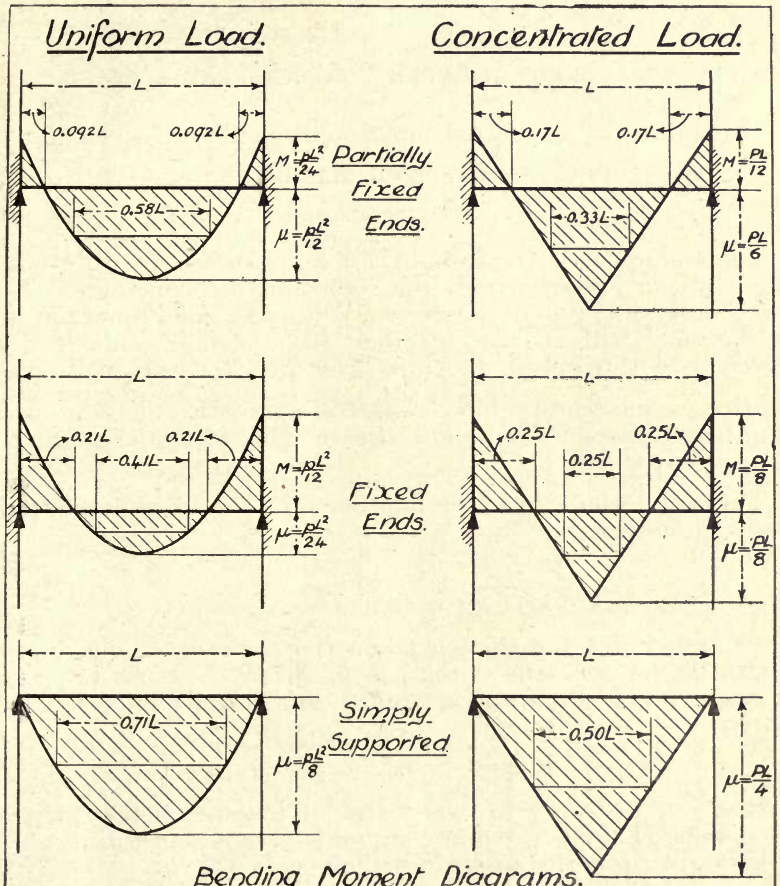
$$\text{Hence the 1st spacing} = \frac{16{,}000 \times 0.22 \times 0.889 \times 21}{\frac{2}{3}\ 25{,}000} = 4'' \ (<6'' \text{ limit})$$

The 4th spacing = 2 x 4 = 8″
The 2nd spacing = 3 x 4 = 12″
The 3rd spacing = $\frac{3}{4}$ of 21″ = 15¼″ (say 15″ max. spacing)

Should the first spacing have been greater than 6″ a smaller diameter stirrup should have been chosen.

VI.—BENT-UP BARS AND PROVISION FOR NEGATIVE MOMENT OVER THE SUPPORT.

When some of the horizontal bars are bent up at convenient points at an angle not greater than 45° but not less than 33°, they assist the stirrups in resisting the diagonal tension. At the same time provision for negative moment can be made by extending these bars horizontally over the support for a length which can be fixed with the assistance of Figure 8. In this figure the variation of the bending moment and the position of the points of inflection are shown for uniform and for concentrated loads, and for both free and fixed ends.



Bending Moment Diagrams.
Figure 8.

Example:

Suppose that the positive moment in a beam requires 6 bars in the middle of the span, that the loading is uniform, and that the beam has both ends securely fixed. Figure 8 shows clearly that in the middle of the span the full section of six bars is required for a length of 0.41L, beyond which, on both sides, three bars can safely be dispensed with. These three bars should be bent up at an angle approaching 45°, and should then be carried horizontally at the top over the support to cover a length of 0.21L on both sides. The steel area over the supports will then be six bars as required to resist the negative moment.

For ordinary cases it is customary to assume,

(1) For restrained beams $M = Wl^2/12$ for negative as well as for positive moment.

(2) For partially restrained beams, or when there is some possibility of poor construction, $M = Wl^2/10$ for negative as well as for positive moments.

When designing by these moments it is essential to provide for an efficient restraint of the beam ends, not only in design but also in construction.

VII.—BOND BETWEEN CONCRETE AND STEEL.

The following additional notation is used in this section,

o = Circumference of one bar.

Σo = Total circumference of bars.

The difference of tension in a bar between two points is transmitted by the bond between the steel and the concrete. The bond stress is thus a function of the shear. Without a good bond the concrete and the steel will not act together and the bars embedded in the concrete will slip before the full stress can be developed.

Stirrups and bent-up bars considerably increase the bond resistance of the tension bars. The bond resistance fails when the elastic limit of the steel is reached.

The unit bond stress, in lbs. per sq. in., of a number of bars stressed in tension is,

$$u = \frac{V}{\Sigma o \times jd} \quad \text{- - - - - - - - - - - -} \quad (21)$$

The safe unit bond stress is taken as 4 per cent. of the compressive strength of the concrete at the age of 28 days. For a 1 : 2 : 4 mix of concrete with a compressive strength of 2000 lbs. per sq. in. after 28 days,

u = 4% of 2000 = 80 lbs. per sq. in.

It is not necessary in every case to investigate the safety of the bars as regards bond, it being commonly accepted as satisfactory when the bars are properly hooked at their ends.

Laps.—An adequate joint between two bars is made by arranging for them to overlap one another for a length called the "grip length."

The grip length is the length of enbedment at which the bond stress and the tensile stress are reached simultaneously.

The formula for grip length is—

$$l = f_s i \div 4u \quad \text{- - - - - - - - - - - - -} \quad (22)$$

here f_s = working tensile stress of the steel at the joint.

i = diameter of bars to be joined.

If f_s is taken at its maximum, where the bending moment is a maximum, which is worse than the usual case then

$$f_s = 16{,}000 \text{ and } u = 80$$

Hence by formula (22), $l = 50i$

Such grip length can be reduced when the ends of the bars are properly hooked.

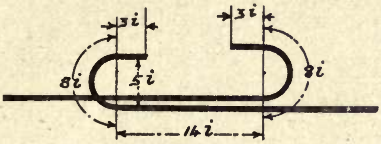


FIG: 9.

The adjoining figure shows the proportions and radius of bending recommended by Considere.

From the figure it is clear that with such proportions and radius of bending the lapping can safely be reduced to : $l = [14 + 2\,(8 + 3)]\,i = 36\,i.$

VIII.—CONCENTRATED LOAD ON CONCRETE SLABS.

By tests it has been proved that if P is the concentrated load applied to a reinforced concrete slab, over a fill, and if,

l = Clear span ; and

E = Total thickness of slab including finishing and filling, then, for the most unfavourable position of the load.

The total bending moment of the slab is,

$M = M_1 + M_2$; in which

M_1 = bending moment due to the dead load,

M_2 = bending moment due to the concentrated load,

$$M_2 = \frac{P}{8} \times \frac{2l - E}{E + \frac{1}{3}l} \quad \text{- - - - - - - - - - - -} \quad (23)$$

In order to simplify the calculations, and make use of a table we take $K = E/l$.

$$\therefore \; M_2 = \frac{3}{8}\, P \times \frac{2 - K}{1 + 3K}$$

$$= CP, \text{ where } C = \frac{3}{8}\,\frac{2 - K}{1 + 3K}$$

Table VIII. supplies the value of the co-efficient C for different values of K.

Table VIII.

K	C
0.05	0.636
0.10	0.548
0.15	0.480
0.20	0.422
0.25	0.375
0.30	0.336
0.35	0.302

Numerical Example.

Design a slab which has to carry a concentrated load consisting of a piece of machinery whose weight, P = 7000 lbs., given,

Thickness of finishing = 1 in.
Thickness of filling (sand) = 6 ins.
Span, l = 6 ft.

Assume a slab thickness of 8″,

Then $E = 8'' + 1'' + 6'' = 15''$.

Ratio $K = E/l = 15 \div 6 \times 12 = 0.20$

Hence by **Table VIII.** we find that $C = 0.422$

$\therefore M_2 = CP = 0.422 \times 7000 = 2954$ lb. ft.

The dead load consists of :—

8″ slab weighing, say, 110 lbs. per sq. ft.
1″ finishing weighing, say, 12 lbs. per sq. ft.
6″ filling weighing, say, 53 lbs. per sq. ft.

Hence total dead load $w = 110 + 12 + 53 = 175$ lbs. p. sq. ft.

$$\therefore M_1 = \frac{wl^2}{10} = \frac{175 \times 6 \times 6}{10} = 630 \text{ lb. ft.}$$

$$M = M_1 + M_2$$

$$= 2954 + 630 = 3584 \text{ lb. ft.} = 43008 \text{ lb. ins.}$$

The simplified formula for slabs derived from formula (4) gives, for a 1 : 2 : 4 mixture of concrete :—

Depth of slab, $d = 0.0316 \sqrt{43008}$
$= 6.57$ inches (say $6\frac{3}{4}$ inches)

Area of steel, $A_s = 0.0058 \times 12 \times 6.57$
$= 0.457$ sq. ins.
($\frac{1}{2}$″ rods at 5″ spacing, give 0.47 sq. ins.)

We can safely fix the thickness of the slab at $7\frac{1}{2}$″ instead of 8″ as being quite ample. The distributing bars for cross reinforcement should be of the same size with a double spacing, i.e., $\frac{1}{2}$″ rods spaced at $2 \times 5 = 10$″.

IX.—SQUARE AND OBLONG SLABS SUPPORTED ON FOUR SIDES.

When a slab is oblong and the length, or long side, L, does not exceed $1\frac{1}{2}$ times the breadth, or short side, B, the loads will be carried by the slab to the four sides. Consequently it must be reinforced in two directions. Rankine's and the French Government formulæ differ slightly, but the resulting difference in the amount of steel is small and unimportant. We shall use the French formulæ applying to them a table simplifying calculations.

The bending moment carried by the short span, B,

$$M_B = \frac{WB^2}{10} \times \frac{1}{1 + \frac{2B^4}{L^4}} \quad \text{or assuming} \quad L = KB$$

$$M_B = \frac{WB^2}{10} \times \frac{K^4}{2 + K^4} = \frac{WB^2}{10} \times (r) \ldots\ldots (24)$$

The bending moment carried by the long span, L,

$$M_L = \frac{WL^2}{10} \times \frac{1}{1 + \frac{2L^4}{B^4}} \quad \text{or assuming} \quad L = KB$$

$$M_L = \frac{WB^2}{10} \times \frac{K^2}{1 + 2K^4} = \frac{WB^2}{10} \times (s) \ldots\ldots (25)$$

W = Total load p. sq. ft.

Table IX.

K	r	s
1	0.333	0.333
1.05	0.378	0.321
1.10	0.423	0.308
1.15	0.467	0.294
1.20	0.509	0.280
1.25	0.549	0.266
1.30	0.588	0.252
1.35	0.624	0.238
1.40	0.658	0.225
1.45	0.687	0.214
1.50	0.717	0.202

In **Table IX.** are given the values of coefficients r and s for various ratios of $\frac{L}{B} = K$, ranging from 1 to 1.50.

Numerical Example.

Design a slab 8′ × 12′ clear spans to carry a uniform load of 130 lbs. per sq. ft.

Assuming the dead load at 50 lbs. per sq. ft. then the total load $W = 130 + 50 = 180$ lbs. per sq. ft.

$B = 8$ ft.

$$\therefore \frac{WB^2}{10} = \frac{180 \times 8^2 \times 12}{10} = 13{,}824 \text{ lb. ins.}$$

$$\text{The ratio } \frac{L}{B} = \frac{12}{8} \quad \therefore \quad K = 1.50$$

$\therefore$ $r = 0.717$ and $s = 0.202$, by **Table IX.**

$$\text{Hence} \quad M_B = 13824 \times 0.717 = 9912 \text{ lb. ins.}$$

$$M_L = 13824 \times 0.202 = 2792 \text{ lb. ins.}$$

Using the usual formulæ for slabs we obtain for :

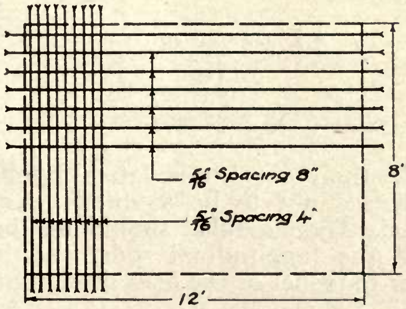
$$\text{The short span,} \quad d = 0.0316\sqrt{9912} = 3.16''$$

$$a_s = 0.0696\,d = .220 \text{ sq. ins.}$$

$$\text{And the long span,} \quad d = 0.0316\sqrt{2792} = 2.08''$$

$$a_s = 0.0696\,d = 0.145 \text{ sq. ins.}$$

The short span fixes the total thickness of the slab at 4″.



The reinforcement required will be :

For short span 5/16″ rods at 4″ spacing ($a_s = 0.230$), and for long spans $a_s = 0.145 \dfrac{2.08}{3.16} = 0.095$ sq. in.

Say 5/16″ rods at 8″ spacing ($a_s = 0.107, > 0.095$)

When the ratio of length to least lateral dimension exceeds 10, reinforced columns should be calculated as long columns.

Short Columns (l/b equal to or less than 10).

The formula to be applied is:—

$$P = P' (1 + (n - 1)p) \quad - - - - - - - - - - \quad (26)$$

in which, P' = safe load for plain concrete core.

= f_c × area of core.

f_c = safe unit compressive stress for plain concrete.

P = safe load for reinforced concrete.

= f_c' × area of core.

f_c' = safe unit compressive stress for reinforced concrete.

p = steel ratio (steel area divided by effective area of column):

The area of core is the effective area of the column after 1½″ has been deducted on each side to provide for the weakening effect of fire.

Tables X., XI., XII., give the safe loads P of short columns of square section with different ratios of steel p; and for three different mixtures of concrete,

Table X. is for a 2000 lb. concrete (1 : 2 : 4) with $f_c = 450$ and $n = 15$.

Table XI. is for a 2500 lb. concrete (1 : 1½ : 3) with $f_c = 565$ and $n = 12$.

Table XII. is for a 3000 lb. concrete (1 : 1 : 2) with $f_c = 675$ and $n = 10$.

Table XIII. gives the values of the safe unit compressive stress for short columns,

$$f_c' = f_c (1 + (n - 1) p) \quad - - - - - - - - - - - \quad (27)$$

for 3 different mixtures of concrete in columns reinforced with vertical rods, but without spiral. The core area required for any load can be immediately obtained from this table since,

$$\text{Area of core} = \frac{\text{Total Load}}{f_c' \text{ (from Table XIII.)}}$$

For round columns,

$$\text{Diameter of column, } D, = \sqrt{\frac{4 \times \text{area of core}}{3.1416}}$$

$$= 1.128 \sqrt{\text{area of core}}.$$

It is important to remember that besides the vertical rods, lateral hoops or bands are essential, although they are not to be counted on as adding to the strength of the columns. Their spacing should not be more than 15 times the diameter of the longitudinal rods, and in no case should the spacing exceed either 18 inches or the least dimension of the core.

Short columns with spiral.

Table XIV. gives the values of the safe unit compressive stress of short columns,

$$f_c' = 1.55 f_c (1 + (n - 1) p) \quad \text{- - - - - - - -} \quad (28)$$

for three different mixtures of concrete, in columns reinforced with vertical rods and with 1% spiral.

$$\text{Area of core} = \frac{\text{Total Load}}{f_c' \text{ (from Table XIV.)}}$$

For round columns,

$$\text{Diameter of column, } D, = 1.128 \sqrt{\text{area of core}}.$$

Table XV. gives the diameter of the spiral bar for different diameters of core, from 12″ up to 36″. The pitch of the spiral is also given so that it shall comply with the specified condition that the spiral reinforcing bar must be not less than 1% of the volume of the column enclosed.

Long Columns (l/b greater than 10)

For long columns Rankine's formula is used :

$$P = \frac{P' (1 + (n - 1) p)}{1 + \dfrac{Kl^2 \times \text{area of col.}}{10{,}000 I}}$$

$$= \frac{\text{Value of } P \text{ for short columns}}{\text{Coefficient of Allowance } R.} \quad \text{- - -} \quad (29)$$

In order to simplify the calculations, the quantity R in the above expression is dealt with as follows,

$$R = 1 + \frac{\text{area of column} \times b^2}{10{,}000 I} \times K \left(\frac{l}{b}\right)^2$$

$$\text{or } R = 1 + S \times K \left(\frac{l}{b}\right)^2 \quad \text{- - - - - - - - - -} \quad (30)$$

$$\text{in which } S = \frac{\text{area of column} \times b^2}{10{,}000 I} \quad \text{- - - - - - - -} \quad (31)$$

For square sections, and taking no account of the steel area,

$$I = \tfrac{1}{12} b^4 \text{ and, area of column} = b^2$$

Hence, for square columns,

$$S = 0.0012$$

$$\text{and } R = 1 + 0.0012 \times K \left(\frac{l}{b}\right)^2 \quad \text{- - - - - - -} \quad (32)$$

Table XVII. gives approximate values of R as computed by formula (32), in which K is taken from **Table XVI.** having regard to the degree of fixing of both ends of the column.

Table XVIII. gives the exact value of S as computed by formula (31) for different types of column section. By inserting the exact value of S in formula (30) we can easily compute the exact value of R.

Table X. — SQUARE COLUMNS.

Safe Loads in lbs. for a Given Percentage of Steel.

Mixture 1 : 2 : 4; $f_c = 450$, $n = 15$.

Side Col. ins.	Side Core ins.	Area Core sq. ins.	$p=0$ (plain)	$p=0.010$	$p=0.015$	$p=0.020$	$p=0.025$	$p=0.030$	$p=0.035$	$p=0.040$
10	7	49	22,000	25,100	26,700	28,200	29,800	31,300	32,900	34,400
11	8	64	28,800	32,800	34,800	36,900	38,900	40,900	42,900	44,900
12	9	81	36,400	41,500	44,100	46,700	49,200	51,800	54,300	56,900
13	10	100	45,000	51,300	54,400	57,600	60,700	63,900	67,000	70,200
14	11	121	54,400	62,100	65,900	69,700	73,500	77,300	81,100	84,900
15	12	144	64,800	73,800	78,400	82,900	87,500	92,000	96,600	101,100
16	13	169	76,600	86,700	92,000	97,300	102,700	108,000	113,300	118,600
17	14	196	88,200	100,500	106,700	112,900	119.100	125,200	131,400	137,603
18	15	225	101,200	115,400	122,500	129,600	136,700	143,800	150,900	157,900
19	16	256	115.200	131,300	139,400	147,500	155,500	163,600	171,600	179.700
20	17	289	130,000	148,300	157,400	166,500	175,600	184,700	193,800	202,900
21	18	324	145,800	166,200	176,400	186,600	196.800	207,000	217,200	227,400
22	19	361	162,400	185,200	196,600	207,900	219,300	230,700	242,100	253,400
23	20	400	180,000	205.200	217,800	230,400	243,000	255,600	268,200	280.800
24	21	441	198,400	226,200	240,100	254,000	267.900	281,800	295,700	309,600
26	23	529	238,000	271,400	288,000	304,700	321,400	338,000	354,700	371,400
28	25	625	281,200	320,600	340,300	360,000	379,700	399,400	419,100	438.700
30	27	729	328.000	374,000	396,900	419,900	442,900	465,800	488,800	512,000
32	29	841	378,400	431,400	457,900	484,400	511.000	537,000	564,000	590,000
34	31	961	432,400	493,000	523,000	554,000	584,000	614,000	644,000	675,000
36	33	1089	490,000	559,000	593.000	627,000	662,000	696,000	730,000	764,000

Table XI. — SQUARE COLUMNS.

Safe Loads in lbs. for a Given Percentage of Steel.

Mixture 1 : 1½ : 3; f_c = 565, n = 12

Side Col. ins.	Side Core ins.	Area Core sq. ins.	$p=0$ (plain)	$p=0.010$	$p=0.015$	$p=0.020$	$p=0.025$	$p=0.030$	$p=0.035$	$p=0.040$
10	7	49	27,700	30.700	32,200	33,800	35,300	36,800	38,400	39,900
11	8	64	36,200	40,100	42,100	44 100	46,100	48,100	50,200	52,100
12	9	81	45,800	50,800	53,300	55,700	58,200	60,700	63,300	65,800
13	10	100	56,500	62,700	65,800	68,900	72,000	75,100	78,300	81,400
14	11	121	68,400	75,800	79,700	83.400	87,100	90,800	94,800	98,500
15	12	144	81,400	90,200	94,800	99,300	103,700	108,100	112,800	117,200
16	13	169	95,500	105,900	111,100	116,500	121,800	127,000	132,400	137,600
17	14	196	110,700	122,800	129,000	135.000	141,200	147,200	153,000	160,000
18	15	225	127,100	141,000	148,000	155,000	162,000	169,000	176,000	183,000
19	16	256	144,600	161.000	169,000	176,000	184,000	192,000	201,000	208,000
20	17	289	163,300	181,000	190,000	199.000	208,000	217,000	226,000	235,000
21	18	324	183,000	203,000	213,000	223,000	233,000	243,000	254,000	264,000
22	19	361	204,000	226,000	237,000	249,000	260,000	271,000	283,000	294,000
23	20	400	226.000	251,000	263,000	276.000	288,000	300,000	313,000	326,000
24	21	441	249,000	276,000	290,000	304,000	317,000	331,000	345,000	349,000
26	23	529	299,000	332,000	348,000	364,000	381,000	397,000	414,000	430,000
28	25	625	353,000	392,000	411,000	431 000	450,000	469,000	490,000	508,000
30	27	729	412,000	457.000	480,000	503,000	525,000	548,000	571,000	593,000
32	29	841	475,000	527,000	554,000	579,000	606,000	632,000	659,000	685,000
34	31	961	543,000	602,000	632,000	662 000	692,000	722.000	752,000	782,000
36	33	1089	615,000	683,000	717,000	750,000	784,000	818,000	853,000	886,000

Table XII. — SQUARE COLUMNS.

Safe Loads in lbs. for a Given Percentage of Steel.

Mixture 1 : 1 : 2; $f_c = 675$, $n = 10$.

Side Col. ins.	Side Core ins.	Area Core sq. ins.	$p=0$ (plain)	$p=0.010$	$p=0.015$	$p=0.020$	$p=0.025$	$p=0.030$	$p=0.035$	$p=0.040$
10	7	49	33,100	36,100	37,500	39,000	40,500	42,000	43,500	45,000
11	8	64	43,200	47,100	49,100	51,000	52,900	54,800	56,800	58,800
12	9	81	54,700	59,600	62,100	64,600	67,000	69,400	72,000	74,400
13	10	100	67,500	73,600	76,600	79,700	82,700	85,700	88,800	91,800
14	11	121	81,700	89,100	92,700	96,400	100,000	103,700	107,500	111,000
15	12	144	97,200	106,000	110,300	114,600	119,000	123,300	127,800	132,200
16	13	169	114,100	124,400	129,400	134,800	140,000	144.800	150,000	155,000
17	14	196	132,300	144,200	150,000	156,000	162,000	168,000	174,000	180,000
18	15	225	151,900	166,000	172,000	179,000	186,000	193,000	200.000	207,000
19	16	256	172,800	188,000	196,000	204,000	212,000	219,000	227,000	235,000
20	17	289	195.100	213,000	221,000	230,000	239,000	248,000	257,000	265,000
21	18	324	218,700	239,000	249,000	258,000	268,000	278,000	289.000	297,000
22	19	361	243,700	266,000	277,000	288,000	298,000	309,000	320,000	331,000
23	20	400	270,000	295,000	307,000	319,000	331,000	343.000	355,000	367,000
24	21	441	298,000	325.000	338,000	351,000	365,000	378,000	392,000	405.000
26	23	529	357,000	389,000	415,000	422,000	438,000	453,000	470.000	486,000
28	25	625	422,000	460,000	479,000	498,000	517,000	536,000	555,000	574,000
30	27	729	492.000	537,000	558,000	581,000	603,000	625.000	648,000	670,000
32	29	841	568,000	619,000	644,000	670,000	696,000	721,000	746,000	772,000
34	31	961	649,000	708,000	736,000	766,000	795,000	824,000	854.000	882,000
36	33	1089	735,000	802,000	835,000	868,000	901.000	933,000	968,000	1,000,000

Table XIII. — COLUMNS.

Values of $f_c' = f_c\ [1 + (n - 1)p]$

(Vertical rods only considered.)

Percentage of Vertical Steel. p	1 : 2 : 4 $f_c = 450$ $n = 15$	1 : $1\frac{1}{2}$: 3 $f_c = 565$ $n = 12$	1 : 1 : 2 $f_c = 675$ $n = 10$
0.010	513	627	736
0.015	545	658	766
0.020	576	689	796
0.025	608	720	827
0.030	639	751	857
0.035	671	782	889
0.040	702	814	918

Table XIV. — SPIRAL COLUMNS.

Values of $f_h = 1.55 f_c\ [1 + (n - 1)p]$

(Vertical rods and 1% spiral.)

Percentage of Vertical Steel.	1 : 2 : 4 $f_{c1} = 1.55 f_c$ $= 700$ $n = 15$	1 : $1\frac{1}{2}$: 3 $f_{c1} = 1.55 f_c$ $= 870$ $n = 12$	1 : 1 : 2 $f_{c1} = 1.55 f_c$ $= 1050$ $n = 10$
0.010	798	965	1144
0.015	847	1013	1191
0.020	896	1061	1239
0.025	945	1109	1286
0.030	994	1157	1334
0.035	1043	1205	1381
0.040	1092	1253	1428

Table XV. — SPIRAL REINFORCEMENT OF COLUMNS.

Diameter of Core in inches.	Pitch in inches.	Diameter of Spiral Bar.
12	2	5/16″
13	$2\frac{1}{8}$	5/16″
from 14 to 17	$2\frac{1}{2}$	3/8″
,, 18 ,, 24	$2\frac{1}{2}$	7/16″
,, 25 ,, 32	$2\frac{1}{2}$	1/2″
,, 33 ,, 36	$2\frac{1}{2}$	9/16″

Table XVI. — Long Columns.

Values of Coefficient K for Different Conditions of Fixing of Ends.

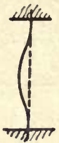
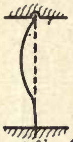
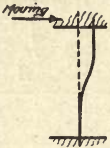
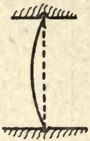
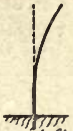
$K = 1/4$	$K = 1/2$	$K = 1$	$K = 1$	$K = 4$
 Both ends fixed in position & direction.	 One end fixed, the other pin-jointed.	 One end fixed in posn: & direction, the other fixed in direcn. only.	 Both ends pin-jointed.	 One end fixed the other free

Table XVII. — Long Columns.

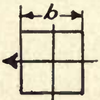
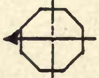
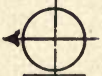
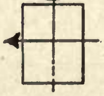
Values of the Coefficient of Allowance, R, in the Simplified Form of Rankine's Formula, for Different Conditions of Fixing of Ends and for Various Ratios of Length to Breadth. $(l/b.)$

K	$R = 1 + 0.0012 \times K\left(\frac{l}{b}\right)^2$ for $\frac{l}{b}$										
	10	12	14	16	18	20	22	24	26	28	30
4	1.48	1.69	1.94	2.23	2.56	2.92	3.32	3.76	4.24	4.76	5.32
1	1.12	1.17	1.24	1.31	1.39	1.48	1.58	1.69	1.81	1.94	2.08
$\frac{1}{2}$	1.06	1.09	1.12	1.15	1.20	1.24	1.29	1.35	1.41	1.47	1.54
$\frac{1}{4}$	1.03	1.04	1.06	1.08	1.10	1.12	1.15	1.17	1.20	1.24	1.27

Table XVIII. — Long Columns.

Values of Factor S, given to Simplify the Calculation of the Exact Coefficient of Allowance, R, in Rankine's Formula.

$$\left(S = \frac{\text{Area of core} \times b^2}{10{,}000 I} \right) \qquad n = 15$$

Type of Section.	Shape of Section.	Percentage of Steel (vertical rods only).				
		0.000	0.010	0.020	0.030	0.040
A		0.00120	0.00108	0.00100	0.00094	0.00090
B		0.00151	0.00144	0.00139	0.00135	0.00132
C		0.00160	0.00155	0.00151	0.00148	0.00145
D		0.00120	0.00108	0.00100	0.00094	0.00090

In the calculation of **Table XVIII.** it should be noted that for reinforced sections the values for S have been obtained from formula (31) by using the following,

(1) Area of core = core area × $(1 + 15p)$ where p is the percentage of vertical steel;

(2) Moment of inertia $I = I_c + nI_s$, which assuming that the distance of the steel from the outer edge of the section is 1/10th of b gives us, for a square section, of side b,

$$I = I_c + nI_s = \tfrac{1}{12}b^4 + 2.4p\,b^4 \text{ where } n = 15.$$

The other sections have been dealt with similarly.

NUMERICAL EXAMPLES OF COLUMN DESIGN.

1. Short Column of Square Section.

Design a square column 10 feet long which has to safely carry a load of 62,000 lbs. The conditions of fixing of the ends are such that $K = \frac{1}{2}$. The concrete mixture is 1 : 2 : 4 for which $f_c = 450$ and $n = 15$.

By **Table X.** a safe load of 62,000 lbs. can be carried by a column 14″ x 14″ with a steel percentage $p = 0.010$. No allowance need be made for lateral bending since

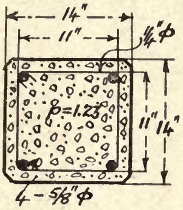


FIG. 11.

$$\frac{\text{Length}}{\text{Breadth}} = \frac{10 \times 12}{14}$$

$$= 8.6 \text{ (which is less than 10).}$$

The necessary area of vertical steel is,

$$a_s = 11 \times 11 \times .01 = 1.21 \text{ sq. ins., say}$$

4 rods $\frac{5}{8}$″ diam. giving 1.23 sq. ins.

For hoops or horizontal bands use $\frac{1}{4}$″ diameter rods spaced 15 x $\frac{5}{8}$ = 9.4, say 9″ apart.

---

2. Long Column of Square Section.

Design a square column 14 feet long which has to carry the same load as above, 62,000 lbs. The fixing of the column ends and the concrete mixture are the same.

By **Table XVII.**, for a ratio of length to breadth of (14 x 12)/14 = 12, (greater than 10) and for $K = \frac{1}{2}$, we see that the Coefficient of Allowance, $R = 1.09$ (approx.). Hence the bearing capacity of the column to safely withstand lateral bending should be 62,000 x 1.09 = 67,580 lbs.

By **Table X.** a safe load of 69,700 lbs., which is the nearest to 67,580 lbs. can be safely carried by a column 14″ x 14″ with a percentage of steel, $p = 0.020$.

Hence steel area $a_s = 11 \times 11 \times 0.020 = 2.42$ sq. ins.,

say 4 rods $\frac{7}{8}$″ diam. givng 2.41 sq. ins.

For hoops $\frac{1}{4}$″ diam. rods spaced 11″ apart may be used, (11″ being less than $15 \times \frac{7}{8}$).

---

3. Long Column of Round Section.

Given a round column 14 feet long, 14 inches in diameter and reinforced with 8 rods $\frac{5}{8}$″ diameter, find the safe load it can carry, without spiral, if the concrete is a 1 : 2 : 4 mix and the condition of the column ends corresponds to $K = \frac{1}{2}$.

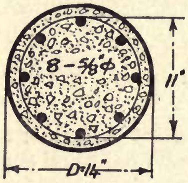


FIG: 12.

The area of a circle being $0.7854D^2$.
Area of core $= 0.7854 \times 11 \times 11 = 95$ sq. ins.
The area of 8 rods $\frac{5}{8}$″ diameter is 2.45 sq ins., hence
Percentage of steel, $p = 2.45/95 = 0.025$ approx.
The ratio of length to breadth is $(14 \times 12)/14 = 12$, hence

$$\left(\frac{l}{b}\right)^2 = 12^2 = 144.$$

The safe load by Rankine's formula is,

$$P = \frac{P'\,(1 + (n-1)\,p)}{R} = \frac{450 \times 95\,(1 + 14 \times 0.025)}{R} = \frac{57{,}712}{R}$$

By formula (30),

$$R = 1 + K.S.\left(\frac{l}{b}\right)^2$$ and from **Table XVIII.**, $S = 0.00150$.

Hence $R = 1 + \frac{1}{2} \times 0.0015 \times 144 = 1.108$.

Consequently, to be perfectly secure against lateral bending, the safe load should be, $P = \dfrac{57{,}712}{1.108} = 52{,}000$ lbs.

It should be noted that instead of computing the value of P' as above, by using formula (26), we could have obtained the same value by applying **Table XIII.** by which, for a 1 : 2 : 4 : mix of concrete with $p = 0.025$ and $f_c = 450$.

$f_c' = f_c\,(1 + (n-1)\,p) = 608$ lbs. from which

$P =$ area of core $\times f_c' = 95 \times 608 = 57{,}760$ which approximates extremely closely to 57,712 lbs. obtained as above.

4. Short Column of Round Section with 1% Spiral.

Find the dimensions and reinforcement required for a round column, with 1% spiral, to carry a load of 724,000 lbs. The ratio of length to breadth of column is less than 10 and the concrete mix is 1 : 2 : 4.

The unit compressive stress of concrete reinforced with vertical bars and with 1% spiral is given by,

$$f_c' = 1.55 \times f_c \; (1 + (n-1)\,p)$$

Assuming a steel percentage of 0.015 for vertical bars and applying either the above formula or **Table XIV.** we find that

$f_c' = 847$ lbs. per sq. in.

Hence, Area of core $= P/f_c' = 724{,}000/847 = 855$ sq. ins.

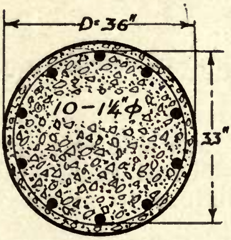


FIG: 13.

Diam. of core $= 1.128 \sqrt{855} = 32.94$, say 33 ins.

Diam. of column $= 33 + 3 = 36$ ins.

Area of vertical steel $= 0.015 \times 855 = 12.825$ sq. ins.
say 10 rods 1¼" diam. giving 12.27 sq. ins.

Spiral : From **Table XV.** for a 33" diam. of core we require a spiral of 9/16" diam. bar at 2½" pitch.

---

5. Long Column of Rectangular Section.

A column is 16.5 ft. long. Its actual load is 62,000 lbs. The least side of the column being fixed at 10 inches, find the length of the other side and the reinforcement required. The fixing condition of the ends corresponds to $K = 1$ and a 1 : 2 : 4 concrete is used.

$$\frac{\text{Length}}{\text{Least Dimension}} = \frac{16.5 \times 12}{10}$$

$$= 20 \text{ approx., hence } \left(\frac{l}{b}\right)^2 = 400.$$

Assuming that the steel percentage, $p = 0.020$ we find from **Table XIII.** that,

$$f_c' = f_c\,(1 + (n-1)p) = 576 \text{ lbs. per sq. in.}$$

and by Rankine's formula

$$62{,}000 = \frac{576 \times \text{area of core}}{R}$$

$$= \frac{(10-3)\,x \times 576}{R}$$

Solving for x,

$$x = \frac{62000 \times R}{7 \times 576} = 15{,}37 \times \text{R}.$$

For a rectangular section (Type F) Table XVIII. gives, for $p = 0{,}020$,

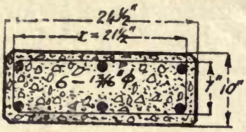


FIG: 14

$$S = 0.001, \text{ consequently,}$$

$$R = 1 + S \times K \left(\frac{l}{b}\right)^2$$

$$= 1 + 0.001 \times 400 = 1.40 \text{ and hence}$$

$$x = 15.37 \times 1.40 = 21.5 \text{ inches.}$$

The length of the Other Side of the Column must then $= 21.5 + 3 = 24.5$ inches.

The Steel area required is given by,

$$a_s = 0.020 \times 7 \times 21.5 = 3.01 \text{ sq. ins.}$$

say 6 rods 13/16″ diam. giving 3.11 sq. ins.

XI.—FOOTINGS OF COLUMNS.

The following example clearly indicates the method of calculation.

Numerical Example.

Design a square footing for a square column 24 inch side carrying a load of 300,000 lbs. The safe bearing power of the soil (sand and clay) is 2 tons per square foot. The concrete mixture to be used is 1 : 2 : 4.

The Area of the Footing is found by dividing the column load, plus assumed weight of footing, by the safe bearing capacity of the soil. Assuming the weight of the footing as 15,000 lbs.,

$$\text{Area of footing} = \frac{300{,}000 + 15{,}000}{2 \times 2{,}240}$$

$$= 71 \text{ sq. ft. approximately.}$$

$$\text{Side of footing} = \sqrt{71} = 8.426 \text{ ft., say 8 ft. 6 ins.}$$

The Minimum Depth is determined by punching shear.

The load producing punching shear is,

$$P \times \frac{\text{footing area} - \text{column area}}{\text{footing area}}$$

$$= 300{,}000 \times \frac{8.5^2 - 2^2}{8.5^2}$$

$$= 282{,}000 \text{ lbs. approximately.}$$

In this computation the dead weight of the footing is not to be included.

The allowable punching shear is $v = 120$ lbs. per sq. in.

Hence the minimum depth of the footing is given by,

$$d = \frac{\text{load producing punching shear}}{\text{perimeter of column} \times v}$$

$$= \frac{282{,}000}{96 \times 120}$$

$$= 24.5 \text{ ins., say 25 ins.}$$

Diagonal Tension develops at a distance from the face of the column equal to the effective depth of the footing.

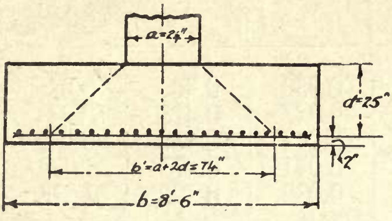


FIG: 15.

Hence side of the effective square, b'

$$= 24'' + 2 \times 25'' = 74''$$

$$= 6 \text{ ft. } 2 \text{ ins. or } 6.17 \text{ ft.}$$

The area outside the same effective square $= 8.5^2 - 6.17^2 = 34.25$ sq. ft.

The total reaction of the soil causing shear is given by,

$$V = 300{,}000 \times \frac{34.25}{8.5 \times 8.5} = 142{,}000 \text{ lbs.}$$

Hence unit shear $v = \dfrac{V}{4 \times b' \times jd} = \dfrac{142{,}000}{4 \times 74 \times 0.889 \times 25}$

$$= 21.6 \text{ lbs. per sq. inch.}$$

As this is less than the limit of 40 lbs. per sq. inch no stirrups are necessary.

The Maximum Bending Moment at the face of the column is,

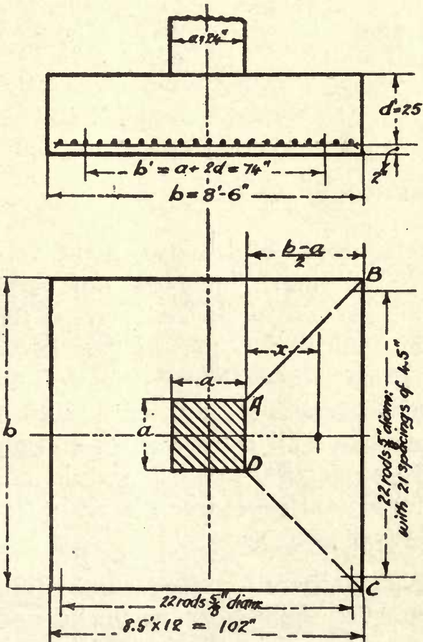


FIG: 16.

$$M = \frac{P}{b^2} \times \frac{1}{4} (b^2 - a^2)\, x$$

in which $\dfrac{P}{b^2}$ = upward unit pressure

$\frac{1}{4}(b^2 - a^2)$ = cantilever area $ABCD$.

and $x = \dfrac{1}{6} \times \dfrac{b - a}{b + a} (a + 2b)$ = lever arm of area $ABCD$ from face AD. By inserting the value of x in the above expression for the bending moment we obtain,

$$M = \frac{1}{4} \times \frac{1}{6} \cdot \frac{(b - a)^2 (a + 2b)\, P}{b^2}$$

By introducting the factor $c = a/b$ we obtain the simplified expression

$$\left.\begin{aligned} M &= \frac{1}{24}(1 - c)^2 (c + 2)\, b.P \\ \text{or } M &= C.P.b. \end{aligned}\right\} \quad (33)$$

The value of the coefficient C is given by **Table XIX.** for different ratios of $c = a/b$.

Table XIX. — COLUMN FOOTINGS (See formula 33).

$c = a/b$	$C = \frac{1}{24}(1-c)^2(c+2)$	c	C	c	C
0.150	0.065	0.275	0.050	0.400	0.036
0.175	0.062	0.300	0.047	0.425	0.034
0.200	0.059	0.325	0.044	0.450	0.031
0.225	0.056	0.350	0.041	0.475	0.029
0.250	0.053	0.375	0.039	0.500	0.026

In the case of our example $c = a/b = 24 \div 8.5' \times 12 = 0.235$

By interpolation we obtain from **Table XIX.** $C = 0.054$

Hence $M = 0.054 \times 300{,}000 \times 102 = 1{,}652{,}400$ lb. ins.

Area of Steel required for the effective width of footing, 74″, is—

$$a_s = \frac{M}{f_s \times jd} = \frac{1{,}652{,}400}{16{,}000 \times 0.889 \times 25}$$

$$= 4.66 \text{ sq. ins.}$$

The area required per foot width is therefore given by

$$a_s = \frac{4.66 \times 12}{74} = 0.760 \text{ sq. ins., say } \tfrac{5}{8}'' \text{ diam. rods spacing}$$

4½″, giving steel area 0.818 sq. ins., greater than 0.760 sq. ins.

We shall place on each side of the footing a set of 22 rods ⅝″ diameter distributed over a width of 21 spacings × 4½″ = 94.5 inches.

XII.—WALL CONSTRUCTION.

1. **Curtain Walls.** These are exterior walls built between the vertical and horizontal framework of a building. They do not carry any load other than their own weight, and are designed to resist the pressure of the wind, which is assumed at 30 lbs. per sq. ft. If we calculated the thickness of an ordinary curtain wall for such pressure we should find a thickness of 4 inches generally sufficient, but for practical reasons the thickness of a curtain wall should never be less than 6 inches. For panels whose area is over 300 sq. ft. it should not be less than 8 inches. The main reinforcement should be placed in the lesser span with stiffening bars in the longer one.

When the walls are built as a separate operation after completion of the framework it is essential to mortise them both vertically and horizontally by leaving slots in the columns and in the floor slabs and beams.

Numerical Example. Design a curtain wall 12 feet high × 25 feet wide between columns. Assume that the wind pressure is 30 lbs. per sq. ft. and that the concrete mixture is 1 : 2 : 4.

The bending moment $M = \frac{1}{10} \times 30 \times 12^2 \times 12 = 5184$ lb. ins.

and the slab depth $d = 0.0316 \sqrt{5184} = 2.28$ ins., say 3 ins.

By placing the reinforcement in the middle, the thickness of the wall will be 2 × 3 = 6 ins.

The steel area of the main reinforcement, placed vertically, is

$a_s = 0.0696 \times 3 = 0.209$ sq. ins., say $\frac{3}{8}''$ rods spacing 6 inches giving 0.221 sq. ins.

The stiffening rods, placed horizontally, should be $\frac{3}{8}''$ rods spacing 12 inches.

2. **Partition Walls.** The thickness of these is governed only by practical considerations, and is generally taken at not less than 1/60th of the height. For instance, a partition wall 12 feet high should have a thickness of not less than 1/60th of 12 × 12 = 2.4 inches, but for practical reasons it should not be less than 4 inches. As with curtain walls, the reinforcement should be placed in both directions in the middle of the slab thickness.

3. **Basement Walls.** These walls are designed as retaining walls resisting the pressure of the earth, which is taken as equal to that of a fluid lighter than water; such weight is assumed as 30 lbs. per cubic foot. Basement walls do not require any particular foundation, being considered as beams supported at both ends by the footings of the columns. For this reason they are also reinforced horizontally at their base as a beam, with a number of straight rods.

Basement walls are usually cast at the same time as the columns, and in such case the vertical reinforcement is added only for stiffening the panels, the main reinforcement being horizontal. Basement walls should not be less than 8 inches in thickness.

Numerical Example. Design a basement wall 10 feet high by 12 feet wide between columns.

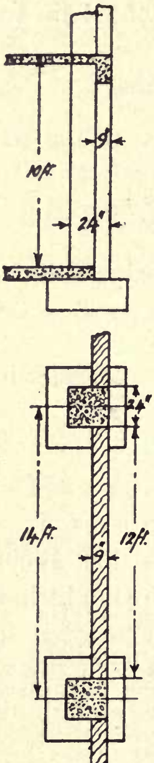


FIG: 17.

The maximum pressure of the fluid, equivalent to the pressure of the earth is, at the bottom of the panel,

$$p = wh = 30 \times 10 = 300 \text{ lbs. per sq. ft.}$$

The bending moment is,

$$M = \frac{1}{10} \times 12^2 \times 12 = 51{,}840 \text{ lb. ins.}$$

The depth of slab is,

$$d = 0.0316 \sqrt{51{,}840} = 7.2 \text{ ins.}$$

The thickness of slab will be taken as 9 inches.

The area of steel (horizontal) is,

$$a_s = 0.0696 \times 7.2 = 0.501 \text{ sq. ins.,}$$

say $\frac{1}{2}''$ rods spacing $4\frac{1}{2}''$ (area = 0.523 sq. ins.).

These rods are to be placed near the inside face of the slab, and are to be carried up for half the height of the wall, say for 6 feet from the bottom. The upper part of the wall, for the remaining 6 feet to the top should be reinforced with the same $\frac{1}{2}''$ rods at double the spacing, i.e., 9 inches. Vertical bars should be added to stiffen the panel, say $\frac{1}{2}''$ bars at 12″ spacing.

XIII. — STAIR CONSTRUCTION.

Two types of design are used for stairways :—

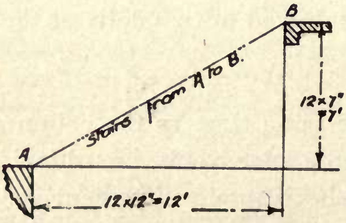


FIG. 17a

1st Type. This consists of an inclined reinforced concrete slab the steps moulded upon its surface. The slab is assumed to be fixed at both ends to floor beams. This type, whilst easily constructed, is heavy, and consequently is not suitable for long spans.

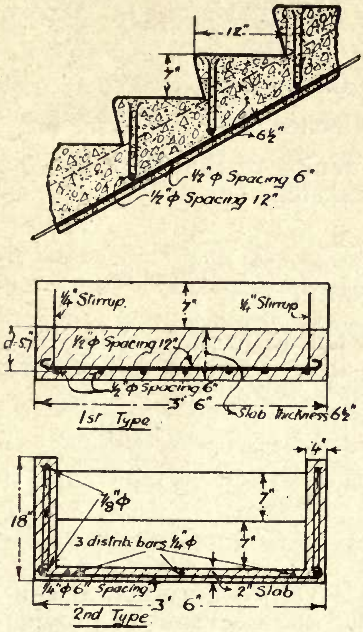


FIG. 18

Numerical Example. Design a stairway 3 ft. 6 ins. wide with a span of 12 ft., consisting of 12 steps including landing. Rise of steps to be 7 ins.

Assume dead and live load to be 190 lbs. per sq. ft., the bending moment

$$M = \frac{1}{10} \times 190 \times 12^2 \times 12$$

$$= 32{,}832 \text{ lb. ins.}$$

$$\therefore \text{ Depth of slab, } d = 0.0316\sqrt{32{,}832}$$

$$= 5.7 \text{ ins.}$$

Hence slab thicknes = 6½ ins.

Area of steel is given by,

$$a_s = 0.0696 \times 5.7 \quad 0.396 \text{ sq. ins.}$$

Use, say, ½″ rods at 6″ spacing, area 0.393 sq. ins.

The secondary reinforcement is shown in the sketch.

2nd Type. This consists of plain concrete steps moulded upon a thin slab supported by side girders in reinforced concrete.

Numerical Example. Design a stairway 3 ft. 6 ins. wide with a span of 14 ft. and having 14 steps. Rise of step to be 7 ins.

Assume the dead and live load to be 220 lbs. per sq. ft.

GIRDERS. Each girder will carry ½ of 220 × 3.5 = 385 lbs. per foot run.

$$\text{Maximum bending moment} = \tfrac{1}{8} \times 385 \times 14 \times 14 \times 12$$

$$= 113{,}190 \text{ lb. ins.}$$

Assuming the dimensions of the girder to be 4″ × 18″, with symmetrical reinforcement in tension and in compression,

$$\text{Lever arm} = 18'' - 4'' = 14''$$

$$\text{Area of steel} = a_s = \frac{113{,}190}{14 \times 16{,}000}$$

$$= 0.510 \text{ sq. ins., say 1 rod } \tfrac{7}{8}'' \text{ diam.}$$

(area 0.601 sq. ins.) in tension, the same in compression.

SLAB. Span of slab = 2 ft. 10 ins. = 2.833 ft.

The dead and live load without girders = 200 lbs. per sq. ft.

Hence maximum bending moment is given by,

$$M = \frac{1}{10} \times 200 \times 2.83^2 \times 12 = 1920 \text{ lb. ins.}$$

Depth of slab, $d = 0.0316 \sqrt{1920} = 1.4$ ins.

Take thickness of slab as 2 ins.

Area of steel, $a_s = 0.0696 \times 1.4 = 0.097$ sq. ins.,

say, $\frac{1}{4}''$ diam. rods spacing 6 inches ($a_s = 0.098$ sq. ins.)

The longitudinal reinforcement for the stiffening of the slab will consist of 3 rods $\frac{1}{4}''$ diameter placed as shown in sketch.

XIV. — SIMPLE TENSION.

Concrete is not assumed to resist any tensile stress. Reinforced concrete is therefore so designed that the area of steel is sufficient to take the whole of the tension.

If T = the total tension

and f_s = the allowable unit tensile stress

$$\text{Steel area } a_s = \frac{T}{f_s}$$

The thickness of the concrete is generally governed by practical considerations, as it must be sufficient to embed the steel and to provide the necessary bond for the overlapping of the bars.

To avoid cracks in the concrete it is necessary to design the steel area with a limiting stress of 12,000 lbs. per sq. inch.

---

1. **Pipes.** Suppose that a water pipe is under a comparatively small external pressure—small enought to be neglected, and that it is subject to an internal pressure due to a head of 40 feet. Taking a factor of safety of 4 it will be necessary to determine the thickness and reinforcement of the pipe for a head, H, of $4 \times 40 = 160$ feet.

The internal pressure, $p = H/2.31 = 70$ lbs. per sq. in. (2.31 is the height in feet of a column of water 1 sq. inch in cross-section and weighing 1 lb.).

For equilibrium the formula is,

$$\tfrac{1}{2} D \times p \times l = f_s \times a_s$$

in which D = diameter of pipe in inches.
$p = H \div 2.31$ = internal pressure in lbs. per sq. in.
l = unit length of pipe = 12 ins.
f_s = 12,000 lbs. per sq. in.
a_s = steel area in 12″ length of pipe.

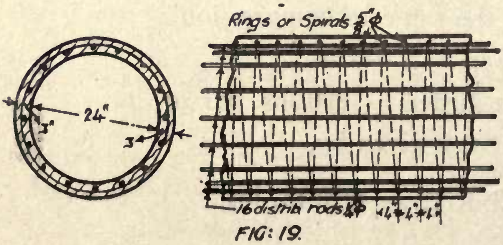


FIG: 19.

Numerical Example.

Design a pipe 24 inches diameter to withstand an internal pressure of 70 lbs. per sq. in., or for a head of 40 feet (see above).

Diameter of pipe, $D = 24''$.
Unit tensile stress, $f_s = 12{,}000$ lbs. per sq. in.
Internal pressure, $p = 70$ lbs. per sq. in.

The area of steel, which is to be divided into a suitable number of hoops or into a spiral, is given by,

$$a_s = \frac{\frac{1}{2}D \times p \times l}{f_s} = \frac{\frac{1}{2} \times 24 \times 70 \times 12}{12{,}000}$$

$= 0.84$ sq. ins., say $\frac{5}{8}''$ diameter, rods spacing $4''$, giving steel area 0.92 sq. ins. (greater than 0.84 sq. ins.), or a spiral of $\frac{5}{8}''$ diameter bar with a pitch of $4''$.

The distributing or longitudinal bars may be calculated by assuming that the portion of the shell between two rings acts as a beam between the rings, supporting a load which is equal to the internal pressure.

Consider a strip of the shell 12 inches wide.

The total pressure on this is, $P = 70 \times 12 \times 4 = 3360$ lbs.

$$\text{The bending moment, } M = \frac{1}{12} P \times \text{spacing}$$

$$= \frac{1}{12}(3360 \times 4) = 1120 \text{ lb. ins.}$$

Placing the reinforcement in the middle of the thickness, which, for practical reasons, can be safely assumed as being 3 inches, the depth, d, = 1.5 ins.

Hence, area of the longitudinal steel is,

$$a_s = \frac{M}{f_s \times jd} = \frac{1120}{12{,}000 \times 0.889 \times 1.5}$$

$= 0.07$ sq. ins.,

say $\frac{1}{4}''$ diameter rods at $6''$ spacing, giving 0.098 sq. ins.

---

2. **Circular Reservoirs.** If p is the pressure in lbs. per sq. foot at any point in a circular tank, and D is the internal diameter in feet, then the tension, T, in a ring one foot high placed at that point will be given by,

$$T = \frac{pD}{2.}$$

At a depth of h feet from the top of a liquid weighing w lbs. per cubic foot the pressure $p = wh$ lbs. per sq. ft., consequently,

$$T = \frac{wh \times D}{2} \quad \text{- - - - - - - - - - - - -} \quad (34)$$

The steel area required in such ring will then be,

$$a_s = \frac{T}{f_s} = \frac{wh \times D}{2 \times f_s} \quad \text{- - - - - - - - -} \quad (35)$$

If the liquid is water, its weight, $w = 62.4$ lbs. per cubic foot.

$$\text{Hence, steel area, } a_s = \frac{62.4 \times h \times D}{2 \times 12{,}000}$$

$= 0.0026hD$ sq. ins., where h and D are given in feet.

Numerical Example. Design a circular tank to contain 90,000 gallons of water.

Volume required = 90,000/6.24, say 14,500 cubic feet.

Assuming an internal diameter of 40 feet and a height of 12 feet,

$$\text{Volume of tank} = \frac{D^2h \times 3.14}{4} = \frac{40^2 \times 12 \times 3.14}{4}$$

$$= 15{,}050 \text{ cubic feet approx.}$$

The dimensions assumed are ample and will be adopted.

The design of the bottom of the tank depends upon the arrangements which have to be made to meet local conditions. We shall consequently determine the dimensions and steel required for the circular vertical slab only.

The pressure per sq. ft. at the base, acting at right angles to the side is,

$$p = wh = 62.4 \times 12 = 748 \text{ lbs. per sq. ft.}$$

At the base the tension in a ring one foot high is

$$T = \tfrac{1}{2}D \times wh = \tfrac{1}{2} \times 40 \times 748$$

$$= 14{,}960, \text{ say } 15{,}000 \text{ lbs.}$$

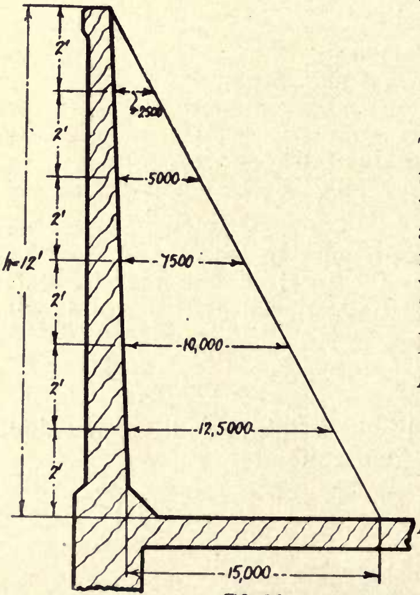


FIG: 20.

STEEL AREA REQUIRED IN BOTTOM RING is,

$$a_s = T/f_s = 15{,}000 \div 12{,}000$$

$= 1.25$ sq. ins., say $\frac{5}{8}''$ rods spacing $2\frac{1}{2}''$ ($a_s = 1.47$).

The pressure decreases from the bottom upwards and becomes zero at the top. By drawing a stress diagram, as shown, we find the stress in each ring. Taking each ring as 2 feet high, we find in the next ring above, that $T = 12{,}500$ lbs.

Hence,

AREA OF STEEL IN 2ND RING is,

$a_s = 12{,}500 \div 12{,}000 = 1.04$ sq. ins., say $\frac{5}{8}''$ rods spacing $3\frac{1}{2}''$ ($a_s = 1.05$).

AREA OF STEEL IN 3RD RING is,

$a_s = 10{,}000 \div 12{,}000 = 0.83$ sq. ins., say $\frac{5}{8}''$ rods spacing $4\frac{1}{2}''$ ($a_s = 0.818$).

The other rings are dealt with in the same way. It is advisable to use the same size of rods for all the rings, gradually increasing the spacing from the bottom upward.

The vertical or distributing bars should be $\frac{1}{2}''$ diameter, spacing not more than 12 inches.

The thickness of the concrete is not governed by values obtained from theoretical calculations, as for pipes, but by practical considerations.

The formula giving the theoretical thickness is as follows,

$$\text{Thickness, } e = \frac{T}{\text{Tensile stress of concrete} \times 12} - \frac{n.\,a_s}{\text{spacing}} \qquad (36)$$

At the base of the tank; $T = 15{,}000$ and $a_s = 0.306$ (one $\frac{5}{8}''$ rod spacing $2\frac{1}{2}''$).

Taking tensile stress of concrete as 120 lbs. per sq. in. and $n = 15$,

$$e = \frac{15{,}000}{120 \times 12} - \frac{15 \times 0.306}{2.5} = 8.6 \text{ ins.,}$$

For practical reasons, however, we shall adopt a thickness of 12ins. at the bottom and 6 ins. at the top.

XV. — RETAINING WALLS.

We shall consider the cantilever wall type only, without buttresses, which is most suitable and economical for ordinary heights up to 18 ft.

THE TOTAL LATERAL PRESSURE at the back of a vertical retaining wall holding up a filling is,

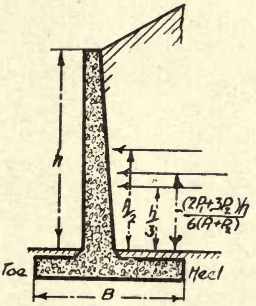


FIG: 21.

$$P_1 = \frac{W}{2} \tan^2\left(45^\circ - \frac{\phi}{2}\right) h^2 \quad - \quad - \; - \qquad (37)$$

in which : W = weight of filling per cubic ft.
h = height of filling.
ϕ = angle of repose of filling.

THE EXTRA LATERAL PRESSURE due to a surcharge of 100 lbs. per sq. ft., such as may be due to a footpath is,

$$P_2 = 100 \times \tan^2\left(45^\circ - \frac{\phi}{2}\right) h \quad - \; - \qquad (38)$$

Such surcharge has not a direct action on the wall. Instead of the surcharge being as above, it may be also due to a load of earth deposited on the filling at its natural angle of repose.

In order to offer the designer a table by which a standard type of cantilever wall can be easily calculated, we have considered a filling consisting of sand whose weight, w = 100 lbs. per cubic foot, and whose angle of repose is 30°.

Inserting these values in formulæ (37) and (38) we obtain :—

$$P_1 = \tfrac{1}{2} \times 100 \times \tan^2 30^\circ \times h^2 = 16.67h^2 \quad - \; - \; - \; - \qquad (37a)$$

$$P_2 = 100 \times 0.333h = 33.33h \quad - \; - \; - \; - \; - \; - \; - \; - \; - \qquad (38a)$$

The pressure P_1 acts at a distance of $\frac{1}{3}h$ from the base of the wall.
and „ P_2 acts at a distance of $\frac{1}{2}h$ from the base of the wall.

The combined pressure $P = P_1 + P_2$ acts at a distance from the base of the wall of

$$\frac{(2P_1 + 3P_2)}{6\,(P_1 + P_2)}\, h \quad - \; - \; - \; - \; - \; - \; - \quad - \; - \; - \qquad (39)$$

Vertical Slab. The maximum bending moment, which is at the base of the wall, where it joins with the foundation slab, is,

$$\begin{aligned} M &= P_1 \times \tfrac{1}{3}h + P_2 \times \tfrac{1}{2}h \\ &= 5.56h^3 + 16.67h^2 \end{aligned}$$

The thickness of the slab, at the base, and the area of steel required are given by using the general formulæ for slabs, which for a 1 : 2 : 4 concrete, are,

$$\text{Thickness} = 2'' + d = 2'' + 0.0316\sqrt{M}$$

$$\text{Area of steel, } a_s = 0.0696d.$$

The shear at the base of the slab does not require special attention as it is very small.

Only half of the vertical steel is usually carried up to the top of the wall as it is not necessary for the full amount to run more than half way up from the base of the wall. The bond of the bars must be effective at the end where the stress is a maximum.

The horizontal bars, otherwise called distributing bars, are generally of the same size as those of the vertical reinforcement, but are given double the spacing.

Foundation Slab. The thickness of the foundation slab is usually taken as equal to the thickness of the vertical slab at its base.

The width does not depend so much upon the nature of the soil as upon the character of the filling at the back of the wall. Assuming a filling of sand, whose angle of repose is 30°, the width, B, of the foundation slab to give security against overturning is given by :—

$$B = 1.5\sqrt{\tfrac{1}{3}h^2 \times \tan^2\left(45^\circ - \frac{\phi}{2}\right)} = 1.5\sqrt{\tfrac{1}{3}h^2 \times 0.333} = 0.50h$$

The best proportions between the toe and heel of the foundation should be found by trial, taking into consideration the conditions of each particular case since some may require a larger heel and some a larger toe.

The bearing capacity of the soil under the foundation slab must be considered. In order to reduce the pressure on the soil it is desirable to bring the resultant force as near as possible to the middle, so as to cut the underside of the foundation slab within the middle, third of the width B.

In Table XX., which follows, the designer will find the main dimensions of cantilever walls from 6 up to 18 feet in height; the filling at the back of the wall is assumed to be sand weighing 100 lbs. per cubic foot with an angle of repose of 30°. The surcharge on top of the filling is taken at 100 lbs. per sq. ft.

Table XX. — CANTILEVER WALLS.

h feet	$P = P_1 + P_2$ lbs.	$\frac{(2P_1 + 3P_2)h}{6(P_1 + P_2)}$ feet	Vertical Slab.			Foundn. Slab.	
			Max. Bendg. Mom. lb.-ins.	Depth d ins.	Thickness t ins.	Width B ft.	Thickness inches.
6	800	2.3	21,600	4.6	6.5	3.0	6.5
8	1333	2.9	46,900	6.9	9.0	4.0	9.0
10	2000	3.6	86,700	9.3	11.5	5.0	11.5
11	2383	3.9	113,000	10.6	12.5	5.5	12.5
12	2800	4.3	144,000	12.0	14.0	6.0	14.0
13	3250	4.6	180,000	13.4	15.5	6.5	15.5
14	3733	4.9	222,100	14.9	17.0	7.0	17.0
15	4250	5.3	270,000	16.4	18.5	7.5	18.5
16	4800	5.6	324,300	18.0	20.0	8.0	20.0
17	5383	6.0	385,300	19.6	21.5	8.5	21.5
18	6000	6.3	453,600	21.3	23.5	9.0	23.5

The proportioning between the toe and heel of the foundation slab is left to the judgment of the designer. When fixed it is necessary to combine the resultant pressure, $P = P_1 + P_2$ with the weight W of the filling at the back of the wall. This can be done graphically and the resultant force R found, both in magnitude and direction, as well as the point where it cuts the underside of the foundation slab. Instead of neglecting the weight of the wall, which is comparatively small, it is usual to include its weight in the cube of the filling.

THE FOLLOWING CASES must be considered :—

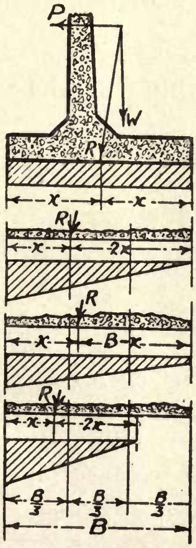


FIG: 22

1ST.—When R cuts the centre of the width B, the unit pressure p, on the ground is uniform, and

$$p = \frac{W}{B}$$

2ND.—When R cuts the base at a distance x from the toe of the wall equal to or greater than $\frac{1}{3}B$, the pressures are,

$$p = \frac{W}{B} + \frac{6W\left(\frac{1}{2}B - x\right)}{B^2} \text{ (at the toe)}$$

$$p' = \frac{W}{B} - \frac{6W\left(\frac{1}{2}B - x\right)}{B^2} \text{ (at the heel)}$$

3RD.—When R cuts the base at a distance x from the toe of the wall equal to or less than $\frac{1}{3}B$,

$$p = \frac{2W}{3x} \text{ decreasing to zero at a distance of}$$

$3x$ from the toe.

THE STABILITY AGAINST SLIDING is ascertained by the ratio P/W, which must be less than the tangent of the angle of friction.

Numerical Example.

Design a cantilever wall to support a sand filling 12 feet high weighing 100 lbs. per cubic foot and having an angle of repose of 30°. The top of the filling is to have a surcharge of 100 lbs. per square foot.

Vertical Slab. Referring to **Table XX.** we at once obtain the following :

The Total Lateral Pressure $P = P_1 + P_2 = 2800$ lbs.

The Lever Arm of the Total Pressure $= \frac{(2P_1 + 3P_2)h}{6(P_1 + P_2)} = 4.3$ feet.

The Maximum Bending Moment $M = 144{,}000$ lb. ins.

Using the usual formula for slabs (1 : 2 : 4 concrete),

Depth, $d = 0.0316 \sqrt{144{,}000} = 12''$, Thickness, at base, $= 14''$.

Area of vertical steel is given by,

$a_s = 0.0058 \times 12 \times 12 = 0.835$ sq. ins. per ft. run,

say $\frac{5}{8}''$ diam. rods spacing $4''$ (giving 0.920 sq. ins.).

These rods are to be placed 2″ from the back face of the wall; half of them should be carried to the top and half should be stopped at a point 6 feet below the top of the wall. The lower end of all rods should be suitably hooked and embedded in the whole thickness of the foundation slab in order to secure a good bond.

From base to top of wall horizontal rods should be placed, at 12″ spacing, to act as distributing and stiffening bars.

The thickness of the wall at the top is selected arbitrarily and for practical reasons should be not less than 6 inches.

Foundation Slab. Again referring to **Table XX.** we obtain :—

Width of Foundation, $B = 6$ feet.

Thickness of Foundation, $t = 14$ inches (same as vertical slab).

The pressure on the soil is caused by the weight, W, of the filling, with which we include the weight of the wall taking its weight per cubic foot the same as that of the filling.

Taking the distance from heel to front face of wall as 4 feet,

$W = 14 \times 4 \times 100 = 5600$ lbs., acting at a distance of 2 feet from the front face of the wall.

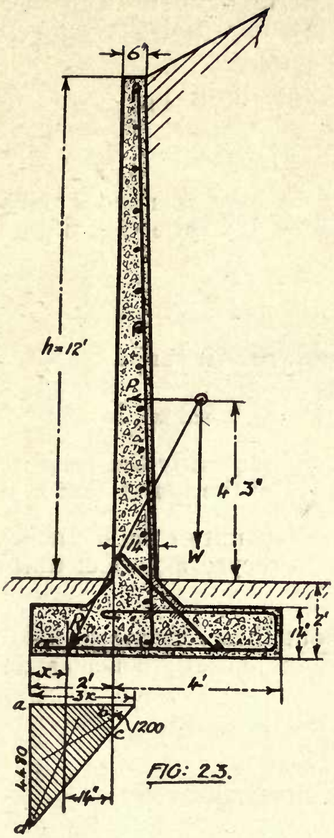


FIG: 23.

The intensity and direction of the resultant force, R (see figure), is obtained graphically, by combining the total lateral pressure, $P_1 + P_2 = P = 2800$ lbs., with the weight, $W = 5600$ lbs.

The resultant R cuts the underside of the foundation slab at a distance of,

$x = 10''$ from the toe; $10''$ being less than $\frac{1}{3}B$, which is $24''$, we fall under CASE 3.

Hence the pressure on the soil at the toe is,

$$p = \frac{2W}{3x} = \frac{2 \times 5600}{2.5} = 4480 \text{ lbs. per sq. ft.,}$$

which is 2 tons per sq. ft.

Taking the soil as hard clay with a bearing capacity of 4 tons per sq. ft., we are within the limit allowed.

Design of the Toe. From the adjoining graphical construction we obtain the bending moment in the toe by multiplying the area of the figure *abcd*, representing the average compression, by the area of the toe and by the distance which the centre of gravity of the figure *abcd* is from the front face of the wall. This distance is 14″ or 1.17′, hence

$$M = \tfrac{1}{2}(4480 + 1200) \times 2 \times 12 \times 1.17 = 79{,}740 \text{ lb. ins.}$$

Using the usual slab formula, for a 1 : 2 : 4 concrete,

$$\text{Depth of Toe, } d = 0.0316\sqrt{79740} = 8.9'', \text{ say } 9''.$$

Thickness $= d + 2'' = 11''$, but we will keep the 14″ previously assumed and therefore the rectified area of steel required will be :

$$a_s = 0.0058 \times 12'' \times 9'' \times 9/12 = 0.470 \text{ sq. ins.,}$$

say $\frac{1}{2}''$ rods, spacing $4\frac{1}{2}''$, giving 0.523 sq. ins.

These rods will run transversely to the wall section for the whole width B.

Shear Stress. The upward pressure on the toe gives the total shear,

Hence $V = \frac{1}{2}(4480 + 1200) = 2840$ lbs.

The unit shear stress is given by,

$$v = \frac{V}{b \times jd} = \frac{2840}{12 \times 0.889 \times 12} = 22 \text{ lbs. per sq. in.}$$

This is well under the limit of 40 lbs. per sq. in. and hence no stirrups are required.

Design of the Heel. The approximate bending moment in the heel is the difference between the overturning moment and the moment in the toe.

Hence $M = 2800 \times 4.3 \times 12 - 79{,}740 = 64{,}740$ lb. ins.

Using the usual slab formula, for 1 : 2 : 4 concrete,

Depth, $d = 0.0316\sqrt{64{,}740} = 8''$, but as we have assumed a thickness of 14″ we shall take the depth of the slab as 12″ instead of 8″ and correspondingly reduce the steel area.

The rectified area of steel required is,

$$a_s = 0.0058 \times 12 \times 8 \times \frac{8}{12} = 0.371 \text{ sq. ins. per ft. run,}$$

say ½″ diam. rods spacing 4½″ giving 0.523 sq. ins.

On the heel the pressure is downwards, consequently the reinforcing bars should be placed 2″ below the top level of the foundation slab.

In the above calculation we have taken no account of the haunches, which are very effective, and are necessary to secure a perfect union between the foundation slab and the wall slab.

In addition to the preceding reinforcement three, or more, longitudinal bars should be placed (as indicated in figure), to act as distributing bars.

XVI.—THE AMERICAN FLAT SLAB CONSTRUCTION.

For a certain class of buildings such as factories, warehouses, garages, etc., with a live load varying from 100 lbs. to 400 lbs. per square foot, the flat slab type of construction is generally more suitable than the beam and girder type, provided that the building consists of at least three bays.

The floor panels are built in sizes ranging, as a rule, from 16 ft. by 16 ft. to 26 ft. by 26 ft. from centre to centre of columns. The reinforced concrete floor slab, which is of uniform thickness, is supported by stiff columns having conical shaped "caps," the object of which is to increase the area of support.

A depressed or "drop panel," which may be made square, rectangular, or hexagonal, is commonly added to reduce the stresses in the slab at its central section, as well as near the columns.

There are two systems in common use: the "**Two-Way System,**" in which the reinforcement is all placed in cross-bands in two directions only, and the "**Four-Way System,**" in which two diagonal bands, as well as two cross-bands, are used. In addition there are short top bands placed over each cross-band, and at right-angles to them.

The formulæ and rules for design are not based so much on theoretical analysis as upon the results of extensive tests carried out in America by important institutions and prominent engineers of that country. The formulæ which have been developed are all ultra-conservative, and give absolutely safe results when used by a competent man.

We shall illustrate the method of design of the "**Four-Way System**," by a numerical example. The computations will be made in detail, and in accordance with the Chicago building code (as amended in 1918), which is considered the best code of rules for flat slab construction. Before dealing with the numerical example we shall set forth the rules to be applied in practice to the design of square panels.

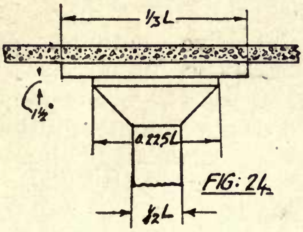


FIG: 24

Columns. The least dimension shall be not less than one twelfth of L, where L is the length of the panel in feet, nor less than one twelfth of the clear height of the column.

Column Caps. Although theoretically unnecessary, it is advisable to reinforce column caps with a few cantilever rods of small diameter, and also with a few circular hoops or rings. It is necessary that such reinforcement be kept as high as possible in the slab, and that it be securely fixed to the vertical rods of the column. This reinforcement is also useful for keeping the slab reinforcement near the top of the slab over the supports.

The diameter of the column cap must be measured where its vertical thickness is at least $1\frac{1}{2}''$ and must be not less than $0.225L$. The slope of the cone should always be such as to make an angle of 45° with the vertical.

Drop Panel. The width must be not less than one third of L. The critical point of maximum stress being at the edge of the drop panel, its depth is determined by consideration of the negative bending moment at that section.

Slab. The minimum thickness of the slab should be determined by the formula,

$t = 0.023L \sqrt{w}$ in which

w = total live and dead load in lbs. per sq. ft., and

t = total thickness of slab in inches.

In no case shall the slab thickness be less than $L/32$ for floors, $L/40$ for roofs, nor less than 6 inches.

The total load on the panel $= W = wL^2$

For purposes of design the panel shall be divided into strips, known as Strip A and Strip B. The widths of Strips A and B shall be taken as $L/2$.

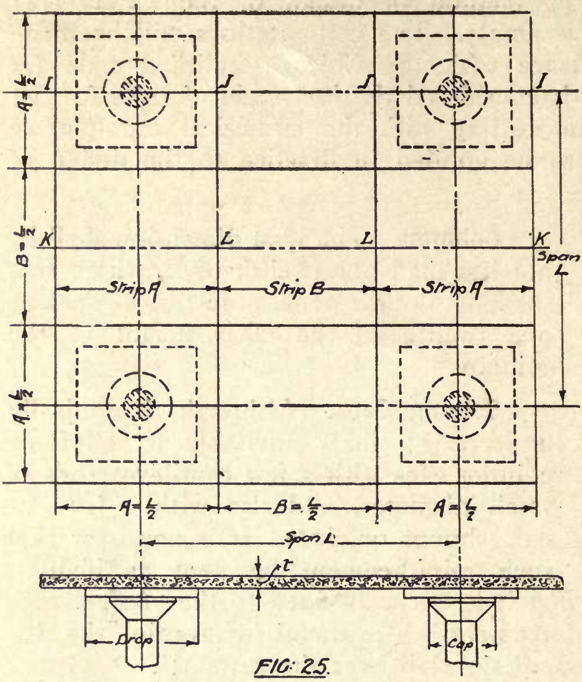


FIG: 25.

It is important that the total bending moment, which is $M = WL$, be adequately distributed between the sections adjoining the columns and the central section.

Interior Panels. The distribution of total moment along the slab is as follows:—

Strip A.

Negative moment at edge of column cap $= -WL/30$.

Positive moment midway between column centres $= +WL/80$.

Strip B.

Negative bending moment on centre line of columns $= -WL/120$.

Positive bending moment in the centre of the panel $= +WL/120$.

Exterior Panels. Where wall panels with drops and capitals are carried by columns and girders built into a wall, as in skeleton construction, the same distribution of bending moment as for interior panels should be used, with this exception. The positive bending moment on Strips A and B, midway between the wall and first line of columns shall be increased 25%.

Wall columns are constructed with caps or brackets, and with drops similar to those in the interior bays, but they are only half the width.

The wall columns should be designed for bending as well as direct compression. It is usual to assume a bending moment of $WL/60$ at the floors and $WL/30$ at the roof. The columns should be designed for this moment, both above and below the slab, and additional reinforcement should be placed as in the adjoining figure to resist it. Such reinforcement should be well tied to the vertical bars of the column.

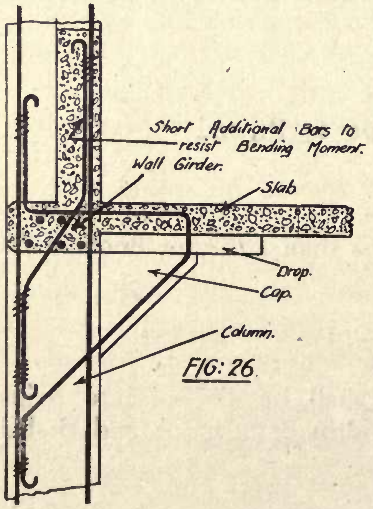


FIG: 26.

Slab Steel. It is common practice to wire together at their intersections, the bars comprising the slab steel. When this is done spacer bars are not required. The top steel has to be kept in the right position by supporting it on small concrete blocks (6″ x 6″ x 6″), about three being placed on each side of the depressed panel.

Rectangular Panels. In no rectangular panel should the length exceed the breadth by more than one third of the latter.

It is safe to consider L as the mean of the two panel dimensions where their difference does not exceed 5%. Where length exceeds breadth by more than 5%, the amount of steel required in Strip A, long direction, both positive and negative, is the same as that required for the same strip in a square panel, whose length is equal to the long side of the rectangular panel. The amount of steel required in Strip A, short direction, both positive and negative, is the same as that required for the same strip in a square panel, whose length is equal to the short side of the rectangular panel.

The amount of steel required in Strip B, positive and negative, is the same as that required for a similar strip in a square panel whose length is equal to the mean of the long and short sides of the rectangular panel.

In no case should the amount of steel in the short side be less than two-thirds of that required for the long side.

Numerical Example of a Typical Interior Panel on the Four-Way System.

Data :—Size of panel = 25 ft. x 25ft.
Live load = 250 lbs. per sq. ft.
Assumed dead load = 150 lbs. per sq. ft.

The Total Load per square foot = 150 + 250 = 400 lbs.

The Total Load on the panel = 400 x 25 x 25 = 250,000 lbs.

Thickness of Slab = $t = 0.023 \times 25 \sqrt{400} = 11.5''$ (greater than $L/32$).

Diameter of Cap = $c = 0.225 \times 25 = 5.625'$, say 5 ft. 9 ins., ($5.75'$).

Diameter of Drop = $D = \frac{1}{3} \times 25 = 8.33'$, say 8 ft. 9 ins., ($8.75'$).

The Chicago ruling (1918) allows a working stress for concrete of 37.5% of the ultimate strength which, for a 2000 lb. concrete (1 : 2 : 4), gives 750 lbs. per sq. in., but we shall adopt lower stresses with the same mix of concrete as follows :—

f_c = 600 lbs. per sq. in., f_s = 16,000 lbs. per sq. in., and n = 15.

Strip A.

The width of this strip, see figure, is $L/2$.

The Positive Moment, which is a maximum at mid-span between columns, is—

$$M = +\frac{WL}{80} = \frac{250{,}000 \times 25}{80} = 78{,}125 \text{ lb. ft.}$$

The effective depth of the slab = $d = 11.5 - 1 = 10.5$ ins.

Hence the steel area required is given by,

$$a_s = \frac{78{,}125 \times 12}{16{,}000 \times 0.889 \times 10.5} = 6.3 \text{ sq. ins.}$$

say 22 rods $\frac{5}{8}''$ diameter (across section *KL*).

These rods are to be placed in rectangular direct bands, whose width is $L/2$ = 12 ft. 6 in.

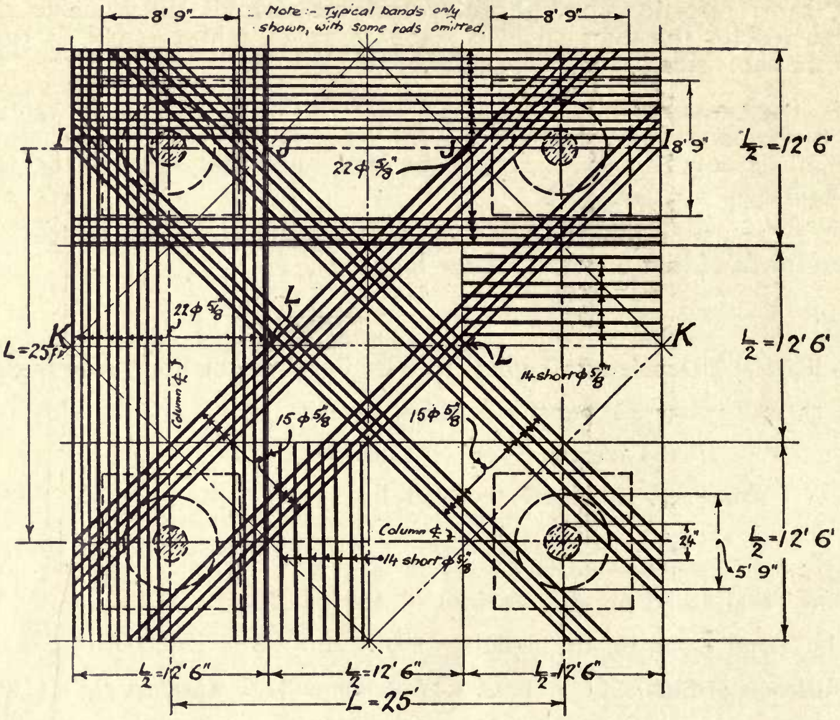


FIG: 27.

Strip B.

The width of this strip, see figure, is $L/2$.

The Positive Moment, which is a maximum in the centre of the panel, is—

$$M = \frac{WL}{120} = \frac{250{,}000 \times 25}{120} = 52{,}000 \text{ lb. ft.}$$

The effective depth of the slab = d = 11.5 − 1.5 = 10.0 ins.

Hence the area of steel required is given by,

$$a_s = \frac{52{,}000 \times 12}{16{,}000 \times 0.889 \times 10} = 4.4 \text{ sq. ins.}$$

say 15 long rods $\frac{5}{8}''$ diameter, placed in each diagonal band (across section *LL*).

The Negative Moment, which is a maximum on the centre line of the columns, is—

$$M = -WL/120 = -52{,}000 \text{ lb. ft.}$$

The effective depth of slab, d = 10.5 ins.

Hence the area of steel required is given by,

$$a_s = \frac{52{,}000 \times 12}{16{,}000 \times 0.889 \times 10.5} = 4.1 \text{ sq. ins. (across section } JJ\text{)}$$

say 14 short rods $\frac{5}{8}''$ diameter. These rods are to be placed in the top of the slab over and at right angles to the direct band.

Strip A.

The thickness of the drop panel, with the thickness of the slab included, is controlled by the negative moment at the edge of the drop panel.

$$M = -\frac{WL}{30} = -\frac{250{,}000 \times 25}{30} = -208{,}300 \text{ lb. ft.}$$

The effective depth of the drop panel is, by formula (4) for slabs,

$$d = \sqrt{\frac{M}{Kb}}$$

in which $K = 83.2$ for $f_s = 16{,}000$, $f_c = 600$ $n = 12$.

The width of the side of the drop panel being $b = 8.75'$,

$$d = \sqrt{\frac{208{,}300}{83.2 \times 8.75}} = 16.9 \text{ ins, say 17 ins.}$$

The total thickness of the drop panel will then be $17 + 1\frac{3}{4} = 18\frac{3}{4}''$.

The steel area required is given by,

$$a_s = \frac{208{,}300 \times 12}{16{,}000 \times 0.889 \times 17} = 10 \text{ sq. ins. (Section } IJ\text{), say 33}$$

long rods $\frac{5}{8}''$ diameter, which are not necessarily divided equally between one direct and one diagonal band.

Distribution of Steel.

The following distribution will be adopted :—

22 rods $\frac{5}{8}''$ diam. in direct bands (Strips A).
14 ,, $\frac{5}{8}''$,, short rods, across direct bands (Strips A).
15 ,, $\frac{5}{8}''$,, in diagonal bands (Strips B).

Out of the 22 rods $\frac{5}{8}''$ diam., which are required on section *KL* to resist the positive bending moment at mid-span of each Strip A, 11 rods should be carried straight in the bottom of the slab and stopped at about 2 feet beyond the centre line of columns.

The remaining 11 rods should be bent up at the quarter point and carried to a point $\frac{1}{4}L = 6$ ft. 3 ins. beyond the same centre line of columns.

By overlapping these 11 rods from each side over the head of the column, in addition to the 15 rods placed diagonally in the panel, the effective area on Section *IJ* to resist the maximum negative moment, at the edge of the column cap, will be $11 \times 2 + 15 = 37$ rods $\frac{5}{8}''$ diam., which is more than the 33 rods required.

It is understood that the 15 diagonal rods, needed on Section *LL*., will all be bent up.

The 14 short rods, required in the top of the slab, over and at right-angles to the direct bands, should be $L/2 = 12$ ft. 6 ins. long.

Shear. The shear stress at the edge of the column cap is given

$$\text{by, } v = \frac{\text{Panel load} - \text{Direct load on cap}}{\text{Effec. depth of drop panel} \times \text{its perimeter}}$$

Hence in our example,

$$v = \frac{250{,}000 - (.7854 \times 5.75 \times 5.75 \times 400)}{3.14 \times 69 \times 17}$$

= 65 lbs. per sq. in. (which is less than the limit of 120 lbs. per sq. in.).

The shear at the edge of the drop panel is obtained in a similar manner, and in our example is given by,

$$\frac{250{,}000 - (8.75 \times 8.75 \times 400)}{4 \times 105 \times 0.889 \times 10.5} = 55 \text{ lbs per sq. in.}$$

which is less than the limit of 60 lbs. per sq. in.

Table for Flat Slab Floors.

Table XXI., which follows, gives for various loadings and spans, both the reinforcement required and the main dimensions of square interior panels of flat slab floors.

For intermediate spans and loads interpolation gives sufficiently accurate figures.

Table XXI.

American Flat Slab Floors.

INTERIOR PANELS — FOUR-WAY SYSTEM.

f_s = 16,000 2000 lb. concrete (1 : 2 : 4).

Square Panels Span	Diam. of Cap		Side of Drop		Slab Thickness	Slab and Drop Thickness	Steel Reinforcement in Each Band		
							Direct	Across Direct	Diagonal
ft.	ft	ins.	ft.	ins.	ins.	ins.	sq ins.	sq ins.	sq. ins.
Live Load = 150 lbs. per sq. ft.									
18	4	0	6	6	6¾	10¾	2.5	1.8	1.9
20	4	6	7	6	7½	12½	3.3	2.1	2.1
22	5	0	8	0	8¼	13¾	4.1	2.6	2.9
24	5	6	8	6	9	15	4.9	3.2	3.5
26	6	0	9	0	10	16	6.0	3.9	4.1
Live Load = 200 lbs. per sq. ft.									
18	4	0	6	6	7	11¼	2.9	2.0	2.1
20	4	6	7	6	8	13	3.9	2.6	2.9
22	5	0	8	0	9	14¾	4.5	3.0	3.2
24	5	6	8	6	9	16¼	5.5	3.5	3.9
26	6	0	9	0	10	17½	6.4	4.3	4.7
Live Load = 250 lbs. per sq. ft.									
18	4	0	6	6	7¾	12½	3.3	2.1	2.3
20	4	6	7	6	8¾	14	4.1	2.6	2.9
22	5	0	8	0	9¾	15¾	4.9	3.2	3.5
24	5	6	8	6	10¾	17¾	6.0	3.9	4.1
26	6	0	9	0	12	19½	6.6	4.7	5.0

BIBLIOGRAPHY.

Faber and Bowie.—Reinforced Concrete Design.

The Concrete Institute (London).—Transactions.

The American Concrete Institute.—Proceedings.

Turneaure and Maurer.—Principles of Reinforced Concrete Construction.

Taylor and Thompson.—A Treatise on Concrete Plain and Reinforced.

George A. Hool.—Reinforced Concrete Construction.

Hool and Johnson.—Concrete Engineers' Handbook.

Prof. Morsch.—Der Eisenbetonban-Seine Theorie und Anwendung.

A. Considère.—Résistance à la compression du béton armé et du béton fretté.

N. de Tedesco.—Traité théorique et pratique de la résistance des materiaux appliquée au béton et au ciment armé.

Coignet et de Tedesco.—Du calcul des ouvrages en ciment avec ossature métallique.

Leon Cosyn.—Traité pratique des constructions en béton armé.

Chicago Building Code.

Sydney Building Regulations.

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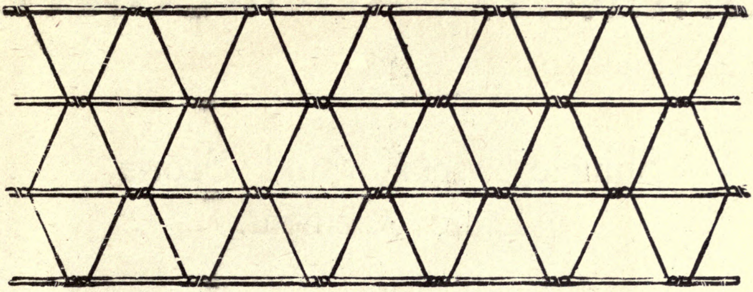

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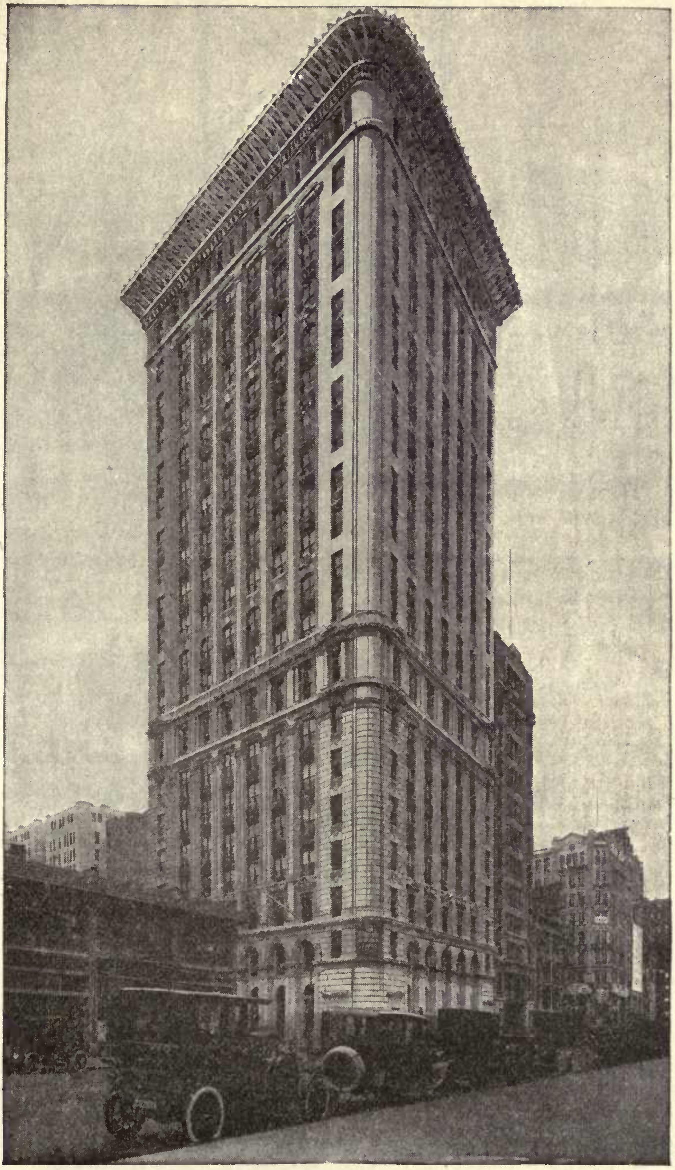

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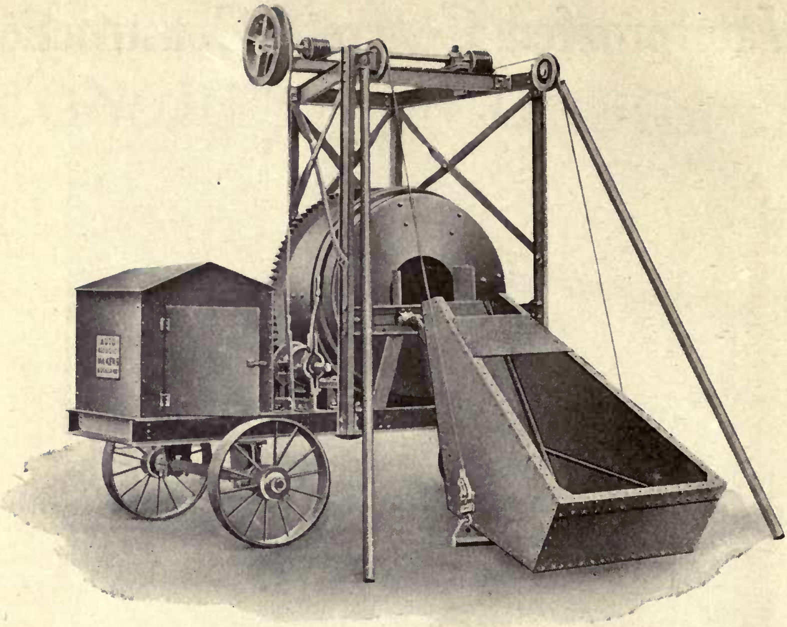

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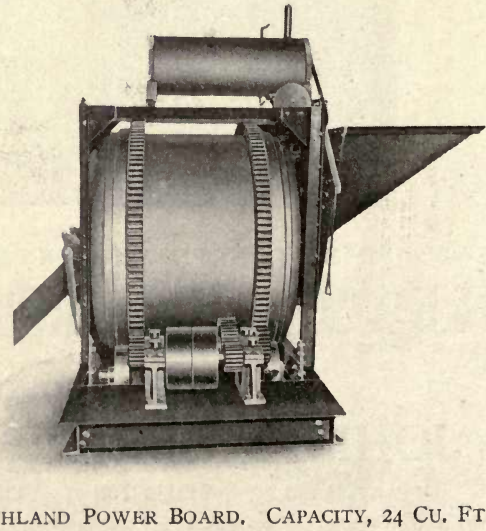

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